American Slavery, Innovation, and Growth

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Job Market Paper

November 2, 2026

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Abstract

Can slavery explain the U.S. South's relative economic decline during the nineteenth century by discouraging innovation? I investigate this question by combining new evidence from historical patent data with a task-based endogenous growth model. I document a number of stylized facts showing low mechanization and patenting in the South before emancipation. Patenting was particularly limited in technologies substituting for enslaved labor. Following emancipation, once access to enslaved labor was no longer possible, patenting in these technologies increased approximately threefold relative to patenting in similar technologies. To explain these patterns, I model coercion as a costless technology under slavery that augments the productivity of coerced labor in routine tasks, making it more competitive against machines. In equilibrium, this leads to low mechanization. Since machines can be improved after their initial invention, weaker incentives to mechanize also slow long-run growth. To assess the quantitative importance of this mechanism, I estimate a two-region model using historical data on the U.S. North and South. The estimated model shows that abolition initially lowers output, but innovation gains eventually dominate, generating substantial long-run gains for the Southern economy. In the most striking case, abolition occurring 60 years earlier would have increased Southern GDP per worker by 43 percent in 1910.

*I am indebted to my advisors Alexander Monge-Naranjo and Russell Cooper for their continuous support, and to Francesco Drago for his excellent suggestions. I also thank Yusuf Agus, Salomé Baslandze, Özlem Bedre-Defolie, Guido Bongioanni, Giancarlo Corsetti, Tuna Dökmeci, Alberto Ferreira, Simon Fuchs, José Garcia, Andrea Gazzani, Marius Grünewald, Leo Kaas, Saverio Lenzi, Alexander Ludwig, David McCarthy, Matthew Mitchell, Lukas Nord, and Paul Rhode, as well as seminar participants at EUI and the Atlanta Fed, for helpful comments, discussions, and assistance with historical data and concepts. All remaining errors are mine.

1 Introduction

In 1774, on the eve of the American Revolution, per capita income in the South was about 50 percent higher than in the North. By 1860, on the eve of the Civil War, the once-richer South had become considerably poorer than the North (Lindert and Williamson, 2012). Lindert and Williamson describe this dramatic shift as a reversal of fortunes in the United States. However, they leave the underlying causes of this divergence largely unresolved.

One possible explanation is the effect of slavery on incentives to innovate. While Northern states had largely abolished slavery following the American Revolution, it remained legal in the Southern states until the end of the Civil War (1861–1865). Moreover, slavery was central to the Southern economy as enslaved workers accounted for roughly half of the Southern workforce (Weiss, 1992). Under this system, labor coercion could increase workers' effort through labor organization (Fogel and Engerman, 1974, 1977), threats of punishment (Fenoaltea, 1984; Acemoglu and Wolitzky, 2011), or intensive supervision (Fenoaltea, 1984; Beckert and Stelzner, 2026). Thus, coercion can make labor relatively cheaper than machinery by extracting greater effort from workers without a proportional increase in labor costs. This, in turn, can weaken firms' incentives to adopt and develop labor-saving technologies. This paper asks whether slavery, through its effects on incentives to mechanize and innovate, can explain the South's relative economic decline.

The impact of slavery on technological change has long been of interest to historians, social scientists and economists. The historian Marc Bloch (1935) argued that the abundance and low cost of enslaved labor in antiquity weakened the economic incentive to adopt expensive labor-saving machinery. Similar arguments concerning antiquity were proposed by Schuhl (1938) and, in part, by Finley (1965).[1] In the context of American slavery, some scholars have also discussed this possible negative effect of slavery on innovation (Genovese, 1965; Fleisig, 1976; Wright, 1978). However, this channel has not been rigorously studied empirically or theoretically. Moreover, it has not been studied as a central mechanism linking slavery to aggregate growth and explaining this major episode of American economic history that led to the economic divergence between the South and the North.

In this paper, I establish and study this mechanism in three parts: (1) empirically, by examining patenting evidence for labor-saving technologies around emancipation; (2) theoretically, by developing a model that connects labor coercion, innovation, and long-run growth; and (3) quantitatively, by calibrating the model to the historical U.S. North and South and conducting counterfactual exercises to quantify the long-run growth effects of slavery and emancipation.

I begin by documenting three stylized facts: before emancipation, the South had fewer patents, lower capital shares, and slower income growth than the North. I then use U.S. Patent Office technology classes to identify inventions designed to replace labor in tasks predominantly performed by enslaved workers, such as cotton picking and sugarcane harvesting. I refer to these inventions as slave-replacing technologies. The comparison group consists of technologies designed to replace labor in tasks predominantly performed by free workers in the North, which I call free-labor-replacing technologies. I find that, before emancipation, there were very few patents in slave-replacing tech-

[1] Schuhl (1938) famously wrote: "L'abondance de la main-d'œuvre servile rend inutile la construction de machines," which translates into English as "The abundance of slave labor makes the construction of machines unnecessary."

nologies. Following emancipation, patenting in slave-replacing technologies increased approximately threefold relative to relative to patenting in free-labor-replacing technologies, compared to the pre-abolition baseline. This finding supports the main mechanism of the paper that coercion discouraged innovation in tasks performed by enslaved workers, and that emancipation subsequently stimulated innovation in these tasks.

To explain these patterns and their implications for growth, I develop a task-based endogenous growth model. Following Acemoglu and Autor (2011), production consists of tasks that can be performed by labor or by machines, once the relevant technology has been developed. Firms choose how to perform each task based on the unit costs of inputs. I model coercion as a costless technology available to firms under slavery. It raises the productivity of low-skill labor, particularly in routine and effort-intensive tasks, consistent with Fenoaltea (1984). By lowering the unit cost of labor in these tasks, coercion makes machines less competitive and their invention less profitable.

The connection between mechanization and subsequent innovation is central to the model's growth mechanism. Technological change takes two forms: inventors can develop a machine for a task still performed by labor, or improve the quality of a machine for a task already mechanized. The latter follows the Schumpeterian quality-ladder framework of Aghion and Howitt (1992). Mechanization therefore opens a task to continued improvements in machine quality. By discouraging the first invention, coercion also closes off these subsequent opportunities for productivity growth. The economy consequently mechanizes fewer tasks and grows more slowly in the long run. The allocation of tasks between labor and machines is therefore central to sustained growth. My model shows how labor institutions shape this allocation by changing the incentives to mechanize.

Removing coercion strengthens these incentives. Tasks that were unprofitable to mechanize under slavery become profitable after abolition, which encourages inventors to develop inventions that can replace labor in those tasks. This provides a theoretical counterpart to the observed increase in slave-replacing patenting after emancipation. The growth gains, however, emerge gradually as new machines must still be invented. As more tasks get mechanized, a greater range of tasks becomes subject to continued technological improvement, which in turn raises productivity growth.

To assess the aggregate importance of this mechanism, I estimate a two-region model of the historical U.S. North and South. The model captures the widening gap in GDP per worker before emancipation and the convergence of regional growth rates afterward. I decompose the South's accumulated income gap relative to the North in 1910 into three components. The lion's share comes from slower growth under slavery before 1865 (38 log points). The immediate productivity loss from removing coercion at emancipation accounts for another 28 log points, while slower growth during the post-emancipation transition adds 7 log points.

I then compare the benchmark abolition in 1865 with a counterfactual in which slavery continues. Under continued slavery, Southern GDP per worker would have been 5 log points higher in 1910 than under abolition. However, as mechanization and subsequent innovation accumulate, abolition leads to higher output in the long run. Southern GDP per worker under abolition overtakes the continued-slavery path around 1918 and is already 23.3 percent higher by 1950. These results show that the long-run gains from abolition eventually outweigh its initial productivity loss. While slavery may have increased current productivity through coercion, as emphasized by Fogel and Engerman (1974),

it also discouraged mechanization and the innovations that sustain long-run growth. These quantified results show the central tradeoff of the paper clearly: once technological change is endogenous, the dynamic costs of coercion can outweigh its static productivity gains.

Finally, I consider alternative abolition dates at 15-year intervals from 1805 to 1880. Had slavery ended in 1805, Southern GDP per worker in 1910 would have been 43 percent higher than under abolition in 1865 and 37 percent higher than under continued slavery. Earlier abolition allows more time for mechanization and subsequent technological improvements, highlighting the importance of earlier abolition for the Southern economy.

Related Literature. The classic literature examines the economic profitability and productivity of slavery. Conrad and Meyer (1958) show that slavery was a profitable investment, while Fogel and Engerman (1977, 1974) document the high measured productivity of slave plantations, particularly under the gang system of labor organization. I use their evidence on plantation productivity to discipline the model, but show that the contemporaneous productivity advantage associated with coercion can lead to lower mechanization and slower long-run productivity growth once technological change is endogenous.

More recent work emphasizes the welfare consequences of emancipation. Hornbeck and Logan (2023) estimate substantial aggregate welfare gains from emancipation despite a decline in measured output, as the reduction in the costs of coercion more than offsets the loss in production. Similarly, Allen et al. (2025) estimate welfare gains of roughly 1,200 percent for formerly enslaved people, which they interpret as a lower bound. I do not attempt to quantify the direct disutility of coercion because I believe that no numerical measure can fully capture the suffering imposed by slavery or the value of freedom itself. My model also predicts an immediate decline in output following emancipation. However, even without accounting for the direct welfare benefits of freedom, removing coercion can generate substantial long-run economic gains by encouraging innovation and accelerating productivity growth. Since utility in the model depends only on consumption, this mechanism can also generate welfare gains through higher consumption.

In a related paper, Fleisig (1976) argues that slavery relaxed agricultural labor supply constraints and thereby made Southern farms more labor intensive. He shows this in a firm-level decision problem and discusses its implications for incentives to invent agricultural machinery. My paper instead builds on the productivity effects of labor coercion and analyzes their dynamic implications in a general-equilibrium growth model with endogenous technological change. This allows me to explicitly characterize the effects of slavery on innovation and growth within the model. Allen et al. (2025) quantify the effects of emancipation in a spatial model in which coercion affects labor supply and the allocation of workers across sectors and locations. They show that emancipation can increase output through the reallocation of labor across sectors and locations. In contrast, my focus is on technological change, and I examine the dynamic gains through productivity growth over time rather than reallocation.

A related literature examines the South's distinctive path of economic development and relatively limited industrialization, emphasizing the structure of local markets, agricultural specialization, investment behavior, and Southern labor-market institutions (Parker, 1970; Engerman, 2014; Wright, 1978; Bateman and Weiss, 1981; Wright, 1986; Ransom and Sutch, 2001). My paper studies the South's relative economic decline through a different channel: the negative impact of slavery on innovation.

Second, the paper relates to the literature on induced innovation and directed technological change. Beginning with Habakkuk (1962), this literature emphasizes that factor scarcity and relative factor prices can shape incentives to develop labor-saving technologies. The modern directed-technical-change literature formalizes how changes in relative prices and market size direct innovation toward different factors and technologies (Acemoglu, 1998, 2002, 2010). A related empirical literature that focuses on historical settings also documents the impact of factor input scarcity on technology adoption and inventive activity (Hornbeck and Naidu, 2014; Hanlon, 2015). My paper is closely related to this literature by studying how coercion changes the effective unit cost of labor and hence incentives for labor-saving innovation, and shows this channel both empirically and in a model.

There are some recent works on innovation and slavery. Olmstead and Rhode (2008) showed that there were ongoing biological innovations that increased the quality of cotton in the American South. More recently, new studies have examined patent data. Mao and Wang (2023) show that labor exploitation weakened the response of agricultural patenting to greater access to finance before the Civil War. Jung and Yoon (2025) find that Southern counties with greater slave concentration in 1860 experienced a relative decline in manufacturing patents after Reconstruction. Rather than using broader agricultural patents, my paper examines the patent data at a more granular level using patent subclasses to identify tasks and using emancipation as a natural experiment. I provide direct evidence that slavery indeed discouraged innovation in technologies that could replace enslaved workers, beyond broad sector comparisons such as those between agriculture and manufacturing.

Finally, the paper builds on the literature on task-based technological change, automation, and growth. Task-based models study how comparative advantage across workers and technologies determines the allocation of tasks and how automation reallocates production from labor toward machines (Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018). The model also draws on the Schumpeterian growth literature, in which continued quality improvements generate productivity growth through creative destruction (Aghion and Howitt, 1992), and on recent work integrating Schumpeterian improvements into task-based automation models (Jones and Liu, 2024; Jones and Tonetti, 2026). My framework builds on these papers while introducing a fixed set of mechanizable tasks that drives long-run growth. This setup is particularly important because it proposes a model in which labor-market effects are not directly corrected by market wage adjustments, even under perfect competition, but can have persistent effects on the economy. With some modifications, it can also be applied to a more general setup where collective bargaining affects incentives to mechanize different tasks, with implications for technological change and long-run growth.

The rest of the paper is organized as follows. Section 2 presents the data, stylized facts, and evidence on innovation. Section 3 develops the model linking coercion to mechanization, innovation, and long-run growth. Section 4 extends the model to the U.S. North and South and presents the estimation from the quantitative model and counterfactual exercises. Section 6 concludes.

2 Empirical Evidence

I begin with three descriptive facts that show the South's relative economic decline and motivate an explanation based on technological change. First, Southern income per capita fell continuously relative

to Northern income before emancipation. Second, patenting activity was concentrated in the North. Third, the capital share was lower in the South and negatively associated with the enslaved population share within the South. Together, these patterns provide an overview of the two regions before abolition and suggest that slavery is associated with lower patenting and capital intensity.

I then provide micro-level evidence using patent subclasses, showing that tasks performed by enslaved workers saw very little patenting before 1860. Using emancipation as an exogenous event that eliminated access to enslaved labor, I show that patenting in these tasks increased three times as much as in comparable tasks that had not relied on enslaved labor.

2.1 Data Sources

The USPTO Historical Patent Data Files (Marco et al., 2015) provide historical patent data with the classes and subclasses of the invention according to the technology to which each invention contributes or which it improves.[2] In addition, I use the census of agriculture data digitized by Haines et al. (2018) that contain variables on farms' machinery values, production quantities for different crops (cotton, tobacco, wheat, etc.), livestock, land size, and the enslaved population at the county level. There are no individual farm-level data; the unit of observation is the county, with farm variables aggregated within each county. I also use historical census of manufactures data (Atack et al., 2006), demographic data (Haines and Inter-university Consortium for Political and Social Research, 2010), and data on the spatial distribution of historical patents from Petralia et al. (2016). I also make use of data from diverse economic history papers for aggregate and regional statistics.

2.2 Stylized Facts

Fact 1: Lower Growth in the South Before Abolition Figure 1 shows the ratio of per capita income in the South to that in the North from 1774 to 1910.[3] This figure reveals a striking pattern. Per capita income in the South was much higher (about 50 percent higher than in the North) on the eve of the American Revolution. However, the initially richer South became relatively poorer over time, and just before the Civil War, per capita income was already well below that in the North. This *reversal of fortune* is driven by the low growth rate of the South relative to the North until abolition in 1865. After an immediate sharp decline following the Civil War and during the turbulent years of the Reconstruction era, the South began growing at a rate similar to that of the North, and their per capita income ratio stabilized in the post-abolition period.[4]

[2]For instance, patent class 56 encompasses harvesting technologies and subclasses from 28 to 51 capture the cotton harvesting technologies, such as Samuel S. Rembert and Jedediah Prescott's prototype of a cotton-picking machine in 1850.

[3]The regional income estimates from 1774 to 1870 are from Lindert and Williamson (2012), while those from 1880 to 1910 are from Klein (2013), who updated the regional income estimates of Easterlin (1960) for these periods. The South is defined as the sum of the states in the South Atlantic, East South Central, and West South Central regions, while the North is the aggregation of all other states. The denominator is the total population, including both free and enslaved people in each region.

[4]These losses between 1865 and 1880 may be due to the direct effects of the war on capital destruction (Goldin and Lewis, 1975; Feigenbaum et al., 2022), the loss of enslaved labor and the collapse of the plantation system (Ransom and Sutch, 2001), and the collapse of the financial system in the South (Bodenhorn and Rockoff, 1992) following the war. I end the period in 1910, just before a number of "great" events: the Great Migration, the Great War, and the Great Depression that occurred between 1910 and 1930 and may have caused other structural changes affecting regional outcomes through different channels. Indeed, the figures in Easterlin (1960) show a slight increase in Southern per capita income relative to that in the

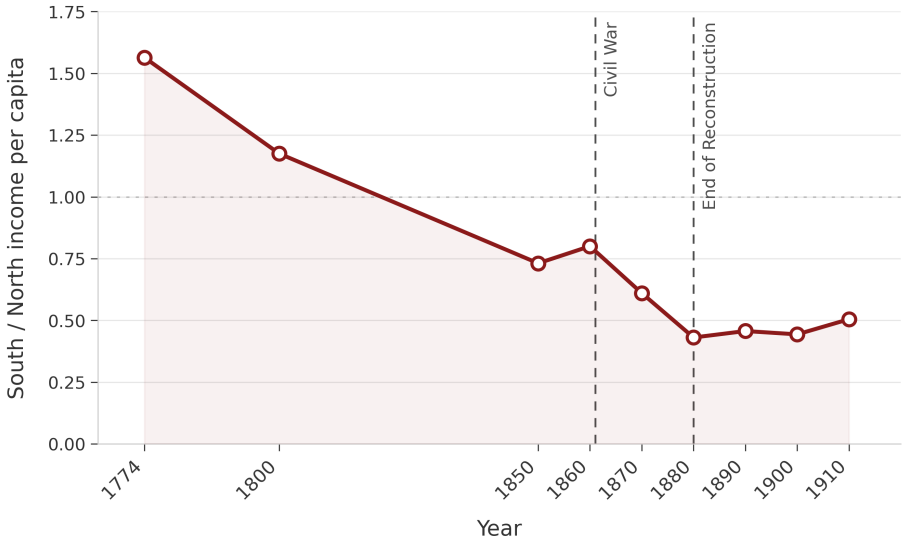


Figure 1: South-to-North Ratio of Income per Capita

The continuous divergence before abolition and its disappearance after abolition motivate the paper's central question: could slavery have slowed Southern growth by discouraging innovation and mechanization? To provide further motivation for this mechanism, the next two facts document the North–South gap in patenting and in agricultural capital shares, since enslaved labor was mostly employed in agriculture.

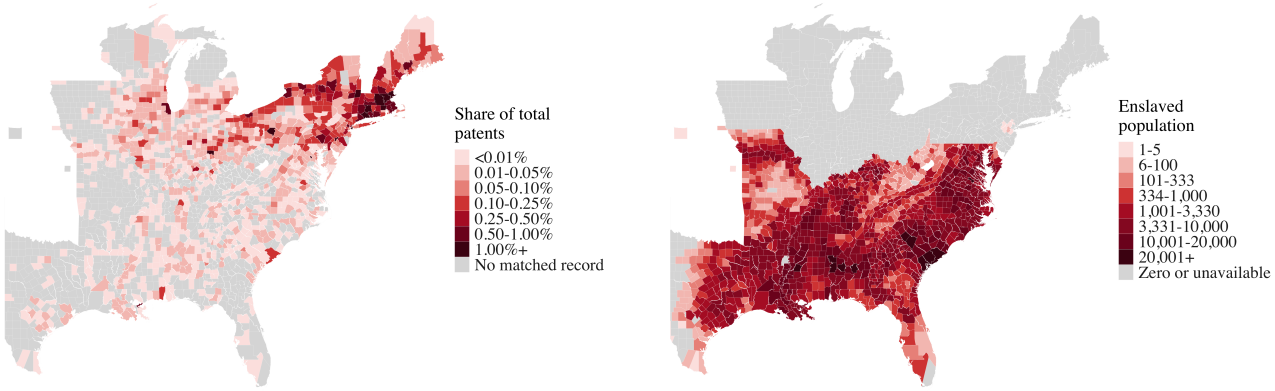
Fact 2: Lower Patenting in the South Before abolition, patenting was concentrated in the North, particularly the Northeast. Using inventor-location data from Petralia et al. (2016), Panel a of Figure 2 reports each county's share of cumulative patent records through 1860. These shares document the geographic concentration of patenting. Panel b shows the distribution of the enslaved population in 1860. The two spatial patterns contrast: the areas with the highest patent shares were in the North, where there was no slavery, while patenting was low in the South, where slavery was concentrated.

Fact 3: Lower Capital Shares in the South and Areas with Larger Enslaved Population Shares There are no county and state level capital share estimates available for the period before the Civil War. Gallman (1972) calculated the capital share of agricultural output for the United States as a whole. Following his methodology, I estimate state and county level capital shares of agricultural output for 1850 and 1860. Appendix Section A.1 describes the estimation procedure. This allows me to compare the capital share of agricultural output across regions and to examine how it varies with the intensity of slavery within the South, providing support for the hypothesis that slavery discouraged mechanization.

Northern agriculture had a higher capital share than Southern agriculture. In 1860, it was 33.1% in the North versus 23.9% in the South (Panel a of Figure 3).[5] In addition, within the South, states

North by 1950, possibly driven in part by the Great Migration from the South to the North.

[5]These averages are calculated from reported state totals, giving each state equal weight. Appendix Figure 15 shows that



(a) County share of total patents

(b) Enslaved population

Figure 2: Patenting and Enslaved Population in 1860

Notes: Both panels use county boundaries as of June 1, 1860 and the same geographic extent. Panel (a) reports cumulative inventor-location records through 1860 as a share of the nationwide total; gray indicates no matched patent record. Panel (b) reports the enslaved population in the 1860 census; gray indicates zero or unavailable population. Sources: Petralia et al. (2016) and Haines and Inter-university Consortium for Political and Social Research (2010).

and counties with a larger enslaved share of the population also had lower capital shares. Panel b of Figure 3 shows this negative relationship across Southern states, and Appendix Table 8 reports the corresponding state- and county-level regressions for 1850 and 1860.

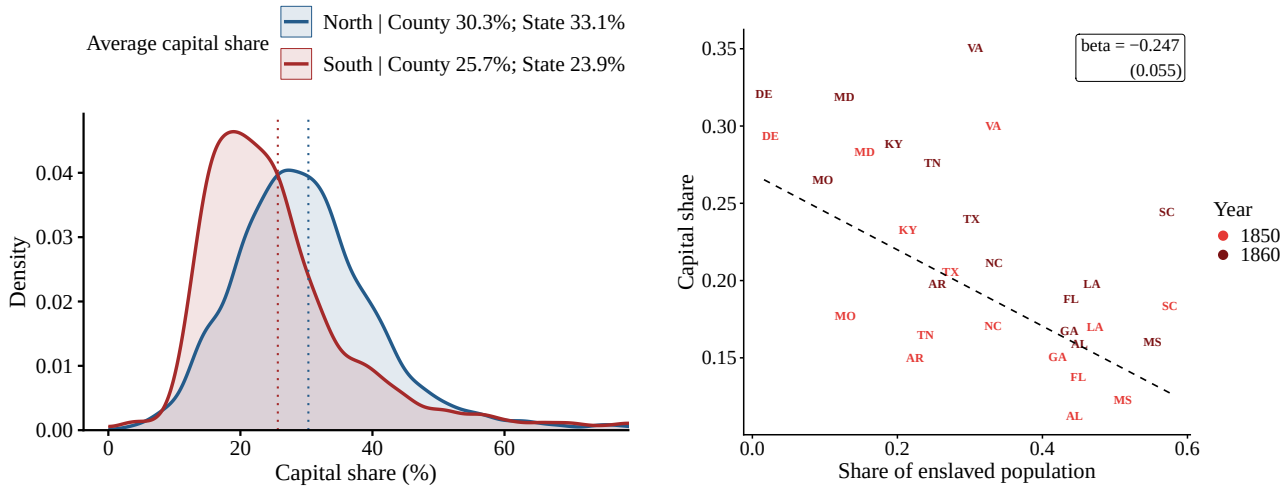
The capital-share measure includes land since land values also incorporate capital investments such as irrigation and drainage infrastructure. Excluding land does not change the direction of these relationships: the North has a higher non-land capital share, and the association with slavery intensity remains negative within Southern states and counties (Appendix Table 8 and Figures 16 and 21). Looking only at farm machinery yields a similar North–South pattern: equipment value relative to the value of agricultural output is higher in the North (Appendix Figure 17). These patterns are consistent with more slave-intensive agriculture relying more on labor and less on capital.

These regional patterns motivate the hypothesis that slavery was indeed associated with lower innovation and mechanization. I next provide micro-evidence from patent classes to examine patenting in technologies associated with tasks performed by enslaved workers and those performed by free workers. I will use abolition as a natural experiment to test whether the removal of coercion increased incentives for labor-saving innovation in tasks previously performed by enslaved workers.

2.3 Micro-Evidence from Patent Classes

Figure 2 documents low levels of patenting in the South. However, the geographical location of an invention does not necessarily coincide with the location of its application. Technologies invented in the North could also be adopted in the South. For example, a cotton-harvesting tool invented in Massachusetts would increase productivity in Georgia, where cotton was produced. Since cotton pro-

this pattern is robust to excluding land from the capital share calculation.



(a) North–South county distributions, 1860

(b) Southern state shares, 1850–1860

Figure 3: Agricultural Capital Shares and Slavery

Notes: Panel (a): counties receive equal weight; dotted lines mark county means. State averages give each state equal weight. Panel (b): the dashed line is the OLS fit with year fixed effects.

duction was overwhelmingly concentrated in the South, such an invention would benefit Southern producers, despite originating in the North. Therefore, I classify technologies according to the agricultural tasks they performed and the labor those tasks employed rather than relying solely on invention location.

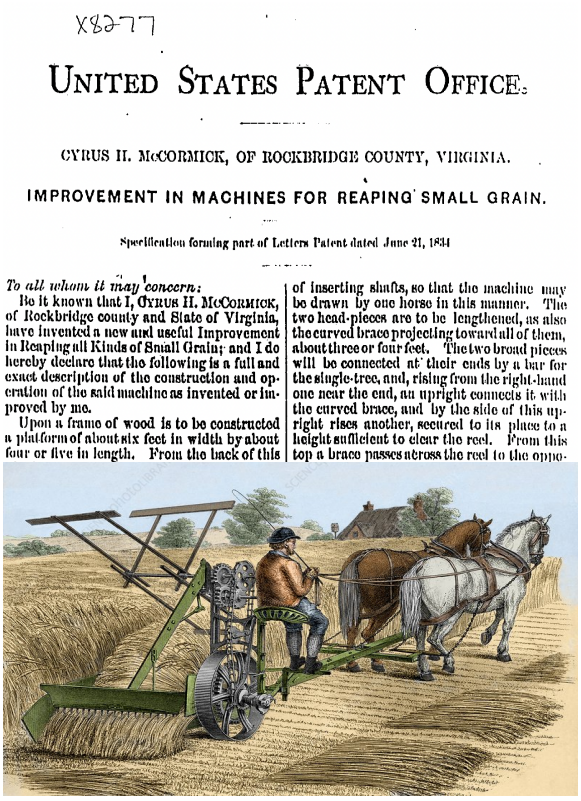
In particular, cotton, tobacco, and sugarcane were major Southern agricultural products whose production relied heavily on enslaved labor (Rhode, 2024).[6] Appendix Figure 22 documents the Southern concentration of these crops. Inventions related to these products can therefore help identify technological developments relevant to the Southern slave economy. However, not all technologies associated with these products were necessarily designed to replace enslaved labor directly. Some inventions improved processing, or other activities that were not necessarily performed by enslaved workers. I therefore distinguish inventions by the tasks they performed and whether those tasks relied on enslaved labor.

A task is defined as a unit of productive activity that contributes to the production of a good or service (Acemoglu and Autor, 2011). In the context of agricultural production, tasks can be organized into broad categories such as digging, planting, cultivation, harvesting, and processing, as illustrated below. These tasks include preparing the soil, planting seeds, tending crops, harvesting, and processing, for example, separating cotton lint from seeds. Each task may require different task-specific skills, tools, and machines.

[6]For instance, Rhode (2024), using the Parker–Gallman farm-level dataset, reports that approximately 95 percent of cotton was produced on slaveholding farms in 1860. The fact that only a narrow set of tasks was performed exclusively by enslaved labor indicates that enslaved workers had a comparative advantage over free workers in these tasks that led to a lower unit cost. If we think of free and enslaved labor as perfect substitutes in performing these tasks, enslaved labor should either have a lower cost than free labor or, for the same cost, higher productivity due to coercion. In either case, this would lead to a lower unit cost for enslaved labor in performing these tasks and make it "cheaper" to employ relative to free labor.



One advantage of looking at the task level of production is that it provides a direct mapping to the patent data. For instance, wheat was the predominant grain of the northern United States (more than 70 percent of it was produced in the North; see Appendix Figure 22). Labor used to harvest wheat was replaced by the horse-powered mechanical reaper invented by Cyrus McCormick in 1831. The McCormick reaper provides a notable example of labor-saving agricultural innovation. David (1966) estimated that self-raking reapers saved approximately 0.36 worker-days per acre harvested.[7]


X8277

UNITED STATES PATENT OFFICE.

CYRUS H. McCORMICK, OF ROCKBRIDGE COUNTY, VIRGINIA.

IMPROVEMENT IN MACHINES FOR REAPING SMALL GRAIN.

Specification forming part of Letters Patent dated June 21, 1834

To all whom it may concern:

Be it known that I, CYRUS H. McCORMICK, of Rockbridge county and State of Virginia, have invented a new and useful Improvement in Reaping all Kinds of Small Grain; and I do hereby declare that the following is a full and exact description of the construction and operation of the said machine as invented or improved by me.

Upon a frame of wood is to be constructed a platform of about six feet in width by about four or five in length. From the back of this of inserting shafts, so that the machine may be drawn by one horse in this manner. The two head-pieces are to be lengthened, as also the curved brace projecting toward all of them, about three or four feet. The two broad pieces will be connected at their ends by a bar for the single-tree, and, rising from the right-hand one near the end, an upright connects it with the curved brace, and by the side of this upright rises another, secured to its place to a height sufficient to clear the reel. From this top a brace passes across the reel to the oppo-

Figure 4: McCormick's reaper and patent

I can trace this innovation in the data through the patent granted to McCormick in 1834 (Figure 4) and related patents that share the same subclass number, which corresponds to grain-harvesting technologies (USPC subclass 194 within USPC class 56, Harvesting). Patent classes and subclasses thus link inventions to specific production tasks; this allows me to identify innovations specifically intended to replace enslaved labor.

For example, in cotton production, processing had already been mechanized well before the rise of cotton production and large plantations in the South, with Eli Whitney's cotton gin patented in 1794. Over time, subsequent innovations further improved this technology. However, cotton harvesting was still performed by enslaved workers. I therefore focus on technologies targeting tasks performed by enslaved workers, which I call slave-replacing technologies. This group includes cotton-harvesting, sugarcane-harvesting, and tobacco-harvesting and processing technologies. The comparison group consists of free-labor-replacing technologies, including grain-harvesting, plow, threshing, and drilling technologies, which primarily replaced free labor in the North.

The analysis sample contains 61 slave-replacing subclasses and 1,382 free-labor-replacing subclasses. Appendix Table 9 details the technology groups and their USPC classes and subclasses. Patent locations support this task-based grouping: before emancipation, slave-replacing patents were substantially more likely to originate in the South, while free-labor-replacing patents were predominantly Northern. Appendix A.4 reports the geographic measures and regressions.

[7]Contemporary accounts also reported substantial labor savings from earlier reapers. For instance, in a letter reproduced in an advertisement for McCormick's reaper, Peyton (1842) reported that the machine could harvest 15 to 20 acres of wheat per day while reducing the labor required from 15 workers to 7, a reduction of more than 50 percent.

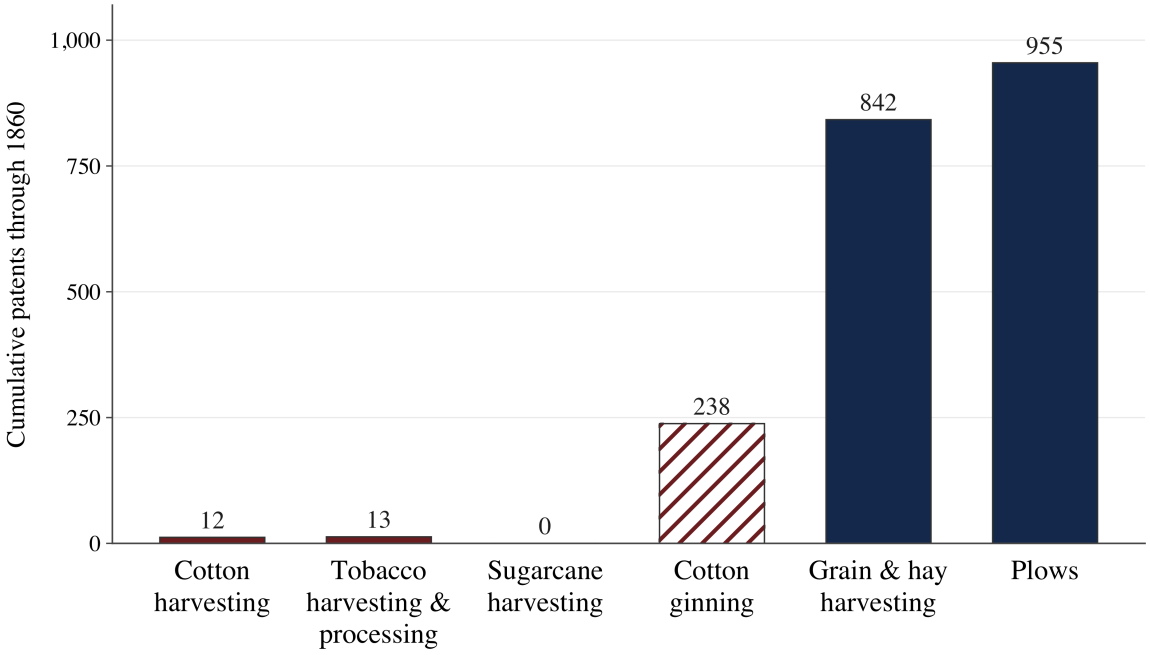


Figure 5: Cumulative Patents by Agricultural Technology, 1860

Notes: Red bars indicate slave-replacing technologies; blue bars indicate free-labor-replacing technologies. The striped bar shows cotton ginning, a processing technology outside the slave-replacing group.

As Figure 5 shows, by 1860, before emancipation, there were very few patents in slave-replacing technologies (red bars), compared with the much larger number in free-labor-replacing technologies (blue bars). Also, there were many more patents for cotton ginning than those for slave-replacing technologies, although fewer than those for free-labor-replacing technologies. This pattern is consistent with the idea that slavery discouraged innovations targeted at replacing enslaved labor, rather than cotton-related innovation as a whole.

If slavery indeed discouraged innovation and kept patenting in these fields low, we should observe a relative increase in patenting for slave-replacing technologies compared with free-labor-replacing technologies following emancipation. To check this, I plot Figure 6 that shows the average patents per subclass for the two groups. Panel (a) shows yearly counts on a linear scale, while panel (b) shows five-year counts on a logarithmic scale. Patenting in slave-replacing technologies starts at a much lower level and increases less than patenting in free-labor-replacing technologies before emancipation. Both groups experience an increase around emancipation, but patenting in slave-replacing technologies seems to continue to grow in subsequent decades, narrowing the proportional gap between them. This descriptive pattern suggests that, following emancipation, patenting in slave-replacing technologies increased relatively faster. However, this requires a more formal analysis using an event-study design, which I present next.

Therefore, I use the abolition of slavery as an exogenous event to test whether it increased innovation in technologies that were targeted at replacing enslaved labor. Neither emancipation nor the Civil War that led to it was fully anticipated by Southern slaveholders. Otherwise, such expectations would have been reflected in the purchase prices of enslaved people. Purchasing an enslaved person was a



(a) Yearly counts, linear scale

(b) Five−year bins, logarithmic scale

Figure 6: Average Patents per Subclass, 1845–1914

Notes: Both panels report arithmetic means across the same subclasses, including zero-count cells. Panel (a) reports patents per subclass per year on a linear axis. Panel (b) reports patents per subclass per five-year bin on a logarithmic axis; bins are labeled by their first year, and 1910 denotes 1910–1914.

long-term investment; thus, if buyers had expected slavery to be abolished within five years, prices should have fallen substantially. In Appendix Figure 24, I plot the purchase prices of enslaved people in both nominal and real terms. These prices, in contrast to possible expectations, increased steadily until the war, suggesting that abolition was not anticipated and priced into the market.

For the event-study analysis, I compare patenting in slave-replacing and free-labor-replacing subclasses before and after 1865. Each observation is the number of patents in a USPC subclass during a five-year bin over 1845–1914, including bins with no patents. To trace changes in patenting before and after emancipation, I estimate the following event-study specification using Poisson quasi-maximum likelihood:

$$\mathbb{E}[\text{Patents}_{ct} \mid c, t] = \exp\left\{\alpha_c + \lambda_t + \sum_{k \neq -5} \beta_k \, T_c D_{kt}\right\}.$$

Since my data has many observed zeros, this approach accommodates zero patent counts well without transforming the dependent variable. Here, Patents_{ct} is the patent count in subclass c during the five-year bin beginning in year t. T_c is the treated group indicator equal to one for the 61 slave-replacing subclasses and zero for the 1,382 free-labor-replacing subclasses (See Appendix Table 9 for classification details). D_{kt} is an event-time indicator equal to one when the five-year bin begins k years relative to emancipation in 1865 ($t - 1865 = k$) and zero otherwise. Subclass fixed effects, α_c, and period fixed effects, λ_t, are included to account for persistent differences in patenting across technologies and changes in patenting common to both groups, respectively.

The coefficients β_k trace how patenting in slave-replacing technologies changes relative to free-

labor-replacing technologies, compared with the last pre-emancipation bin, 1860–1864. Event time k is measured in years relative to 1865, in five-year steps, so the omitted bin has $k = -5$ and the first post-emancipation bin has $k = 0$.[8]

Figure 7 shows that slave-replacing patenting increases relative to free-labor-replacing patenting from 1870 onward and stays much higher than the pre-emancipation baseline throughout the post-emancipation period. There are no visible pre-trends, although the treated group has very sparse observations for this period. The two-way fixed-effects Poisson difference-in-differences estimate for the post-emancipation period is 1.10. This implies an approximately threefold increase in slave-replacing patenting relative to free-labor-replacing patenting, compared with the pre-emancipation period ($\exp(1.10) \approx 3.0$). Both the timing and direction of the increase are consistent with the main mechanism of this paper. Once slavery was abolished and access to enslaved labor was removed for firms, incentives to innovate in tasks previously performed by enslaved workers strengthened, and consequently, innovation and patenting increased more than in similar tasks performed by free labor.

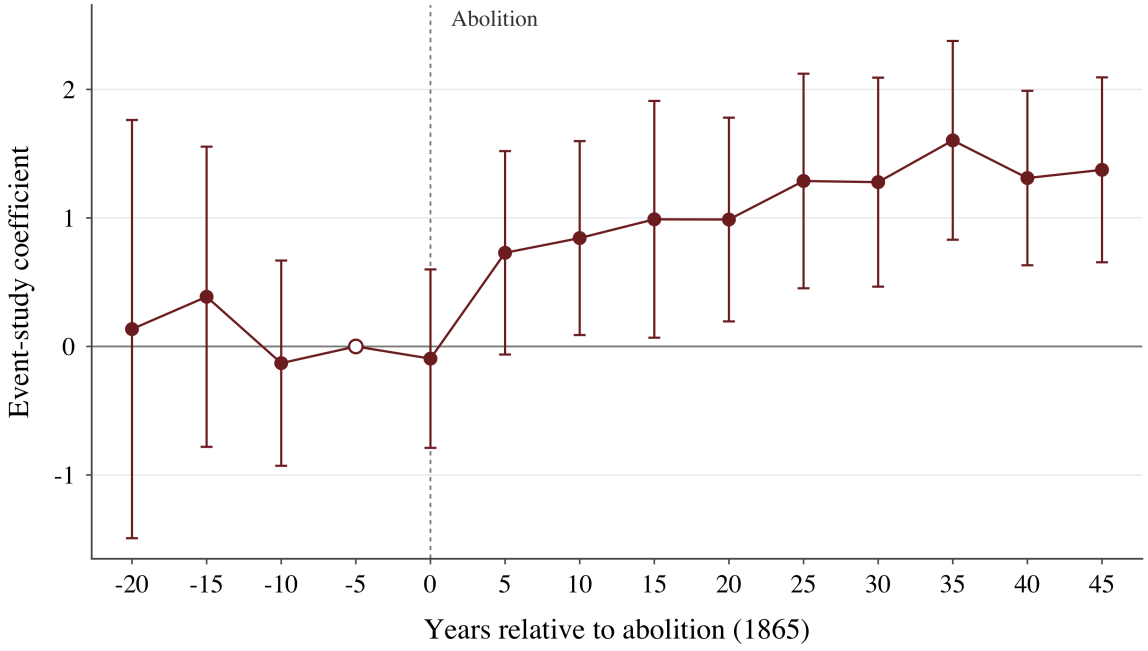


Figure 7: Effect of Abolition on Slave-Replacing Innovation: Event-Study Estimates

Notes: Vertical bars show 95% confidence intervals with standard errors clustered by subclass. The hollow point marks the omitted reference bin, 1860–1864, normalized to zero.

I did a number of robustness checks on this result using alternative comparison groups, bin lengths, reference periods, and placebo tests. Comparisons with broader agricultural patents and with grain/hay-harvesting technologies, as well as alternative bin lengths to assess sensitivity to the aggregation of patent counts over time, all yielded a similar significant increase. Also, alternative prewar reference periods show that the result does not depend on using some Civil War years in the baseline bin (Appendix Figure 25). Moreover, I conducted patent class assignment placebo tests to assess whether the

[8] A positive coefficient indicates a relative increase in slave-replacing patenting. The corresponding percentage change is $100[\exp(\beta_k) - 1]$. Standard errors are clustered by subclass to allow patenting within a technology to be correlated over time.

estimated increase is unusually concentrated in the actual slave-replacing group. Robustly, the actual estimate exceeds all 1,000 random-assignment estimates (Appendix Figure 26). Similarly, using earlier dates as a placebo test to assess whether comparable increases are detectable before emancipation yields no significant increase in slave-replacing patenting (Appendix Figure 27). Together, these robustness checks support the specificity of the response to the technology group and the post-emancipation period.

Furthermore, in Appendix Section A.8, using census data, I check whether the capital share of Southern farms also increased after emancipation. I compare Northern and Southern counties near the 1860 slave–free-state border, where similar crop mixes make the counties comparable. I find that, in the 25 km border sample, the DiD estimate implies that Southern farm capital shares increased by 13.9% relative to those in nearby Northern counties after emancipation. The DDD estimate implies a 34.5% increase in the South–North ratio of farm capital shares relative to the corresponding change in manufacturing capital intensity. Together, these findings support the view that slavery held back mechanization: after emancipation, Southern counties growing crops similar to those of nearby Northern counties increased their farm capital shares relative to their Northern neighbors. Thus, these findings support the mechanism behind the patenting response in Figure 7: when firms are less willing to adopt labor-saving machines, forward-looking inventors have weaker incentives to develop them.

Overall patenting also increased in the South relative to the rest of the United States. In the full US sample, the ratio of Southern to non-Southern patent counts rose from about 11 percent in 1845–1864 to 15 percent in 1865–1914, a relative increase of roughly one-third. Excluding the Civil War years, 1861–1865, reduces this relative increase to about 6 percent; once new western states are excluded, the relative increase is around 14 percent (Appendix Section A.7).

3 Model

The empirical evidence documents low innovation in slave-replacing technologies before emancipation and an increase in innovation in these technologies following emancipation. To assess the implications of this for long-run development, I develop a general-equilibrium model linking labor coercion, mechanization, and endogenous technological change. The model establishes how a contemporaneous productivity advantage from coercion in certain tasks can coexist with weaker incentives to mechanize and slower productivity growth, even as wages and prices adjust. It also provides predictions for the response of mechanization and innovation to emancipation that guide the interpretation of the empirical evidence.

I then extend the baseline model to a two-region economy and discipline it using historical U.S. data. This quantitative framework connects the task-level response to emancipation to the evolution of Southern and Northern income and allows me to evaluate how the timing of abolition affects the accumulation of technology and Southern income.

3.1 Modeling Slavery through Labor Coercion

In the model, slavery is considered the extreme case of labor coercion, in which owners hold full legal property rights over enslaved workers. Owners can rent out their enslaved labor to other firms, as often happened in the antebellum United States, and can appropriate the rental income (Evans, 1962; Martin, 2009; Rönnbäck, 2021). As Martin (2009) estimates, an enslaved worker was three to five times more likely to be hired out than to be sold in a given year. For hiring firms, the relevant cost was not simply the subsistence payment but the overall cost of hiring an enslaved worker from the owner. This total cost could include not only the rental payment but also subsistence costs for food and shelter and, in some cases, supervision, surveillance, and other expenses. Once these costs were added up, hiring enslaved labor would not necessarily be cheaper. In particular, the rental rate was negotiated between the slave owner and the hiring firm, rather than between the enslaved worker and the firm. In the latter case, the firm's strong bargaining power would, of course, imply a very low labor cost. Throughout the paper, I will not consider these costs separately, but rather focus on the total cost incurred by firms when hiring enslaved labor, which I refer to as the rental rate. Indeed, Allen et al. (2025), using Margo's dataset on the hiring of free and enslaved labor at U.S. forts, show that once they control for ration payments to free workers, they find no markdown for enslaved labor relative to free labor for the same occupations. This finding suggests that observed labor-cost differences between free and enslaved workers mainly reflect the allocation of free workers to higher-paying, cognitive jobs and enslaved workers to lower-paying tasks (such as fieldwork) on average.

With these costs in mind, firms compare the cost of employing enslaved labor to perform a task, given its productivity, with the cost of employing free labor to perform the same task, given its respective productivity. It therefore matters which tasks firms hire enslaved labor to perform and whether they can extract high output relative to the amount they pay. This also helps explain why enslaved labor was concentrated in a narrow set of sectors rather than employed in every task in the antebellum United States, as might be expected if enslaved labor were much cheaper for every task.

For simplicity, both types of labor are supplied inelastically in a perfectly competitive setup. This might seem odd, especially given the peculiar labor institution of slavery. As noted above, the rental agreement is made between the slave owner and the hiring firm, so there is no reason to assume that the hiring firm must have monopsony power over slave owners. Indeed, Naidu (2020) argues that American slavery was closer to the textbook competitive model of labor markets than free labor was. Because enslaved workers could not freely choose or leave their jobs, hiring firms could not exploit workers' preferences for particular employers to exercise monopsony power, while the extensive slave-hiring market provided a highly elastic supply of labor.[9] Thus, the household side can be modeled as in the standard case, with a fixed supply of each type of labor. Coercion operates on the production side, while labor costs are determined in equilibrium by firms' demand for each type of labor relative to its supply.

The essence of slavery in this paper is the increase in productivity through coercion. Once firms hire enslaved labor, they can coerce workers at no cost to increase their work intensity. In the model, greater

[9]Here Naidu (2020) refers to the labor supply elasticity facing individual firms, rather than aggregate labor supply elasticity. A perfectly elastic labor supply at the firm level is therefore fully consistent with an inelastic aggregate labor supply in a perfectly competitive market.

work intensity is captured by higher task level productivity. However, this increase in productivity is uneven across tasks. This coercion-induced productivity increase is greater in routine tasks that rely mainly on physical effort and smaller in complex tasks that require cognitive skills, judgment, or care.

This distinction is closely related to Fenoaltea's classification of tasks as effort-intensive and care-intensive (Fenoaltea, 1984). For example, coercion can increase labor productivity through greater physical effort in tasks such as cotton picking, hoeing, or breaking and moving stones in mining, while its effect is more limited in tasks such as accounting, where literacy, judgment, and careful execution are more important than physical effort. Fenoaltea (1984), drawing on Finley (1992), gives the example of enslaved workers employed mainly in mining in ancient Athens and later in Rome. He argues that the threat of whipping and punishment could be used to extract greater physical effort in moving stones. As Fenoaltea (1984, pp. 641–642) notes: "the breaking and moving of stones is as effort-intensive an activity as can be imagined... unfree labor will have a special advantage in such activity... and tend to displace from there free labor that cannot be similarly driven." Here, Fenoaltea already distinguishes tasks in which coerced labor has a Ricardian comparative advantage over free labor. But he also notes that this advantage would be weaker or absent in what he calls care-intensive tasks. This can also help explain the sorting of enslaved labor into different industries, such as mining in the ancient world and plantation agriculture in the American South.

The high productivity of enslaved workers in field work in the antebellum United States is also well documented. The contemporary observer Frederick Law Olmsted recorded a Maryland planter's claim that enslaved workers "at hoeing and any steady field-work" would "do two to their [whites'] one" (Olmsted, 1856, p. 10).[10] Empirically, Fogel and Engerman (1974, 1977) show that slave plantations had higher productivity than Northern free farms and Southern non-slave farms. They connect this higher productivity to greater work intensity under coercion. Similarly, Olson (1992), drawing on plantation records, estimates fewer working hours for enslaved workers than for free farmers in the North and argues that this supports the view that the observed higher productivity was due to greater work intensity per hour rather than longer working hours, as enslaved workers could be forced to work harder. More recently, Naidu (2020), using production function estimates, finds higher estimated marginal products of labor on slave plantations compared to free farms in the antebellum South.

Thus, the model is largely motivated by this evidence on the effects of coercion on productivity. It works as follows. Slavery gives firms access to coercion, which I assume they can apply at no additional cost. Coercion raises labor productivity most in routine tasks, where machines also have a comparative advantage and are more likely to replace labor. Rental rates adjust in equilibrium. The law of one price implies a uniform rental rate across tasks for each type of labor, even though the productivity gains from coercion vary across tasks. Coercion increases productivity most in low-index tasks, while the wage (or rental rate) adjusts based on the average productivity increase across tasks. This makes coerced labor particularly cheap in terms of its unit cost in the routine tasks where machines also have a comparative advantage. Thus, coerced labor is "cheaper" in the sense of having lower unit costs in those tasks. Therefore, even with perfectly competitive markets for both free and enslaved labor, the uneven productivity gains from coercion can reduce the effective unit cost of enslaved labor relative to

[10]There are other passages in his work in which planters similarly describe enslaved labor as more effective than free workers in most plantation tasks when properly supervised.

machines in routine tasks.

3.2 Preferences

Time is continuous. The economy admits a representative household with the following log-utility form:[11]

$$U(t) = \int_t^\infty \exp\left(-\rho(s-t)\right) \ln C(s) ds.$$

There are two types of labor in the economy: high-skill labor (H) and low-skill labor (L). The representative household is endowed with fixed measures H and L, both of which are supplied inelastically to firms. The total labor endowment is therefore

$$N = H + L.$$

The interpretation of low-skill labor changes according to whether coercion exists or not. The model can be interpreted as follows: under a free-labor system, low-skill workers supply their labor competitively, whereas under coercive labor institutions, their labor can be coerced and rented to firms by its owner. The representative household receives the income generated by low-skill labor in both regimes. Accordingly, $w_{L,t}$ is interpreted as the competitive wage under free labor and as the rental payment appropriated by the owner of coerced labor under slavery. Thus, the institutional regime, whether slavery exists or not, changes the interpretation of $w_{L,t}$, but not the household budget constraint. High-skill labor receives the wage $w_{H,t}$ under both regimes. Therefore, the household budget constraint is given by

$$\dot{\mathcal{A}}_t + C_t \leq r_t \mathcal{A}_t + w_{H,t} H + w_{L,t} L,$$

with the standard no-Ponzi-game condition $\lim_{t\to\infty} \mathcal{A}_t \exp(-\int_0^t r_s \, \mathrm{d}s) \geq 0$, where $\mathcal{A}_t = \int_0^{I_t} V_t(i) \, \mathrm{d}i$ denotes the value of the household's asset holdings, and $V_t(i)$ is the value of the innovating firm at task i. This maximization problem yields the usual Euler equation:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{1}$$

[11] I use a representative household to keep the household side simple, while coercion operates on the production side. The main purpose of the paper is to study the effects of coercion on aggregate growth, not the welfare effects on different households. And for this purpose, having heterogeneous households does not change the BGP conditions, aggregate growth rate, or mechanization results of this model. However, the representative-household setup does not allow me to study how welfare effects differ across households. In Appendix F, I therefore consider two separate households. One supplies high-skill labor, owns firms, and saves, and also owns enslaved labor under coercion. The other supplies low-skill labor, does not save, and consumes all the income it receives. Under coercion, the low-skill household receives only a share of the rental income generated by its labor, which I interpret as the subsistence share. The rest goes to the owner household. Under free labor, the low-skill household receives its full labor income. This allows me to study how consumption and welfare effects are distributed across the two households. Although the transition equations change, the quantitative effects are small, and the aggregate paths and parameter estimates remain almost unchanged after re-estimation. Thus, I keep the household side simple in the main model, as is common in growth papers, such as Acemoglu et al. (2018).

3.3 Production Technology

Production takes place over a unit continuum of tasks indexed by $i \in [0,1]$. Tasks are ordered by complexity: low-indexed tasks are relatively routine and effort-intensive, whereas high-indexed tasks are more complex and skill-intensive. The final good is competitively produced by combining these tasks according to

$$\ln Y_t = \int_0^1 \ln y_t(i)\,\mathrm{d}i, \tag{2}$$

where $y_t(i)$ denotes the output of task i. The price of the final good is normalized to one.

Each task can be produced using one of three perfectly substitutable inputs: high-skill labor, low-skill labor, or the capital input. Before specifying task production, I first describe how the productivity of these inputs varies across tasks. These task-specific productivity differences generate comparative advantage across tasks and will determine the equilibrium assignment of inputs characterized later in the model.

Labor productivity and coercion. All workers have a common task-productivity schedule $\gamma(i) > 0$, which is continuous and strictly increasing in i. High-skill labor has productivity at task i,

$$A_H\gamma(i), \qquad A_H > 0,$$

whereas low-skill labor has baseline productivity at task i,

$$A_L\gamma(i), \qquad 0 < A_L < A_H.$$

Thus, absent coercion, high-skill labor has an absolute productivity advantage at every task. Under institutions that permit labor coercion, the productivity of low-skill labor is augmented by a task-specific coercion technology $c(i)$, so that its effective productivity becomes

$$A_L c(i)\gamma(i),$$

with

$$c(i) > 1, \qquad c'(i) < 0.$$

Coercion therefore raises the productivity of low-skill labor, but its effectiveness declines with task complexity in high-indexed tasks. The productivity gain is largest in routine, effort-intensive tasks and becomes weaker in tasks for which output depends more heavily on complex cognitive skills.[12] When coercion is not institutionally available, I set

$$c(i) = 1 \qquad \text{for all } i.$$

[12]The assumption that coercion raises measured productivity through an effort margin is consistent with Acemoglu and Wolitzky (2011), who show in a principal–agent framework that coercion can increase worker effort and output. The reduced-form term of the coercion technology, $c(i) > 1$, captures this productivity effect. The declining schedule of the coercion technology, $c'(i) < 0$, incorporates the idea that coercion gets less and less effective in high-indexed tasks that require skill, judgment, or care, as emphasized by Fenoaltea (1984). Moreover, historical evidence on the higher productivity of slave plantations compared to free farms in the antebellum U.S. South is documented by Fogel and Engerman (1974, 1977).

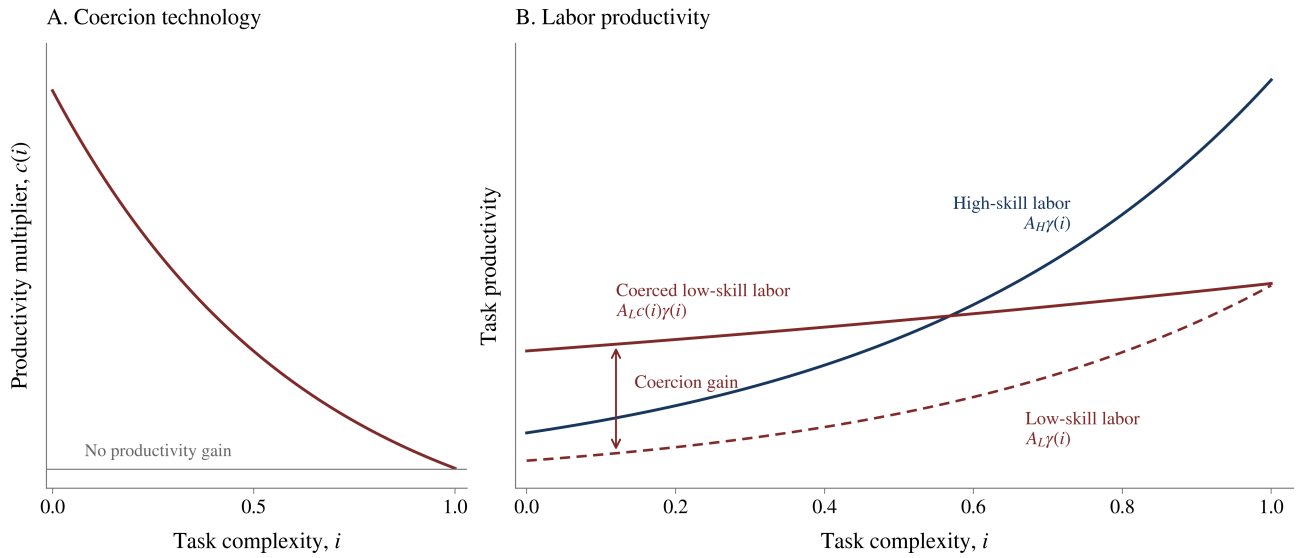


Figure 8: Coercion and Task-Specific Labor Productivity

Finally, I assume that

$$\frac{\mathrm{d}}{\mathrm{d}i}\left[c(i)\gamma(i)\right] > 0.$$

Hence, although the productivity advantage generated by coercion declines with task complexity, the absolute productivity of coerced labor remains increasing in the task index. Figure 8 illustrates these productivity schedules.

Machine technology. In addition to the two labor technologies, tasks that have been mechanized can be performed using a capital input. Let $I_t \in [0,1]$ denote the mechanization frontier, so that the machine technology is available for tasks $i \in [0, I_t]$, with time-varying productivity $q_t(i)$. A newly mechanized task i starts with initial capital productivity $q_0(i)$. I assume that $q_0(i)$ is strictly decreasing in i, so that routine tasks are initially more amenable to mechanization than complex tasks. In particular,

$$q_0'(i) < 0, \qquad \lim_{i \downarrow 0} q_0(i) = \infty.$$

The second condition is an Inada-type boundary condition.[13]

Once a task has been mechanized, its capital productivity $q_t(i)$ can subsequently increase through endogenous quality improvements. By contrast, the underlying labor productivity schedule $\gamma(i)$ is time invariant. Technological change at the task level therefore operates through two margins: the expansion of the set of mechanized tasks, summarized by I_t, and improvements in the quality of machine technologies already in use.[14]

[13] With finite positive factor prices, it ensures that capital productivity is sufficiently high near $i = 0$ for the capital input to be used on a positive measure of tasks in equilibrium.

[14] The assumption that $\gamma(i)$ is time invariant should be interpreted as an approximation to relatively slow improvements in human productivity compared with machine productivity. Consider harvesting. Holding fixed the tools and procedure used to perform the task, the productivity of manual labor changes comparatively little over time. The large historical gains in-

The capital input can be made using high-skill labor. Producing one unit of the capital input requires $\psi > 0$ units of high-skill labor. Hence, the marginal production cost of the capital input is $\psi w_{H,t}$.

Task Production function. Now we can define the task production function. Recall that these three inputs are perfect substitutes within a task. Therefore, the available task-production technologies can be written compactly as

$$y_t(i) = \begin{cases} A_L c(i)\gamma(i) l_t(i) + A_H \gamma(i) h_t(i), & \text{for all } i \in [0,1], \\ A_L c(i)\gamma(i) l_t(i) + A_H \gamma(i) h_t(i) + q_t(i) k_t(i), & \text{for all } i \in [0, I_t]. \end{cases} \tag{3}$$

where $l_t(i)$ and $h_t(i)$ denote low- and high-skill labor employed in task i, respectively, and $k_t(i)$ denotes the capital input employed in task i. The first line shows that every task can be produced using either type of labor, whereas the second line shows that mechanized tasks can additionally be produced using the capital input.

Comparative advantage across tasks. The productivity terms of each input imply an ordering of comparative advantage across tasks. Under coercion, the productivity of low-skill labor relative to high-skill labor is

$$\frac{A_L c(i)\gamma(i)}{A_H \gamma(i)} = \frac{A_L}{A_H} c(i),$$

which is strictly decreasing in i. Coercion therefore gives low-skill labor its greatest relative productivity in low-indexed tasks compared to high-skill labor. Similarly, the productivity of the capital input in a newly mechanized task relative to low-skill coerced labor is

$$\frac{q_0(i)}{A_L c(i)\gamma(i)}.$$

Because $q_0(i)$ is strictly decreasing whereas $\gamma(i)c(i)$ is strictly increasing, this ratio is also strictly decreasing in the task index. Machines therefore have their greatest relative productivity in the lowest-indexed tasks. Together, these monotonicity properties generate a natural ordering of comparative advantage across tasks. Once factor prices and resource constraints are introduced, this feature gives rise to an equilibrium task allocation in which the capital input is used in low-indexed tasks, coerced low-skill labor in intermediate tasks, and high-skill labor in high-indexed tasks.

stead come from changes in the procedure used to perform the task: manual harvesting was replaced by mechanical reapers, and later by tractors and combine harvesters, whose productivity continued to improve as machine quality advanced. In the model, the transition from labor to machines corresponds to mechanization, while subsequent improvements in the machines used to perform the task are captured by growth in $q_t(i)$. This interpretation is consistent with Jones and Tonetti (2026), who emphasize the reallocation of tasks from slowly improving labor to rapidly improving machines.

3.4 Resource Constraints

The supplies of high- and low-skill labor are fixed at H and L, respectively. Low-skill labor is allocated directly across tasks, so that

$$\int_0^1 l_t(i)\mathrm{d}i = L.$$

High-skill labor can be employed either directly in task production or in producing the capital input. Since producing one unit of the capital input requires ψ units of high-skill labor, its resource constraint is

$$\int_0^1 h_t(i)\mathrm{d}i + \psi \int_0^{I_t} k_t(i)\mathrm{d}i = H.$$

Let R_t denote total expenditure on R&D. The aggregate final-good resource constraint is

$$Y_t = C_t + R_t. \tag{4}$$

3.5 Firm Optimization and Task Allocation

The competitive final-good sector chooses task inputs to minimize expenditure subject to (2). The Cobb–Douglas structure implies equal expenditure across tasks:

$$p_t(i)y_t(i) = Y_t \qquad \text{for all } i \in [0,1], \tag{5}$$

where the price of the final good is normalized to one.

Within each task, the three inputs are perfect substitutes. Their relative production costs therefore determine which technology is used. The unit costs of coerced low-skill labor, high-skill labor, and an available machine are

$$\underbrace{\frac{w_{L,t}}{A_L c(i)\gamma(i)}}_{\substack{\text{Coerced low-skill labor}\\ \text{unit cost at task } i}} \qquad \underbrace{\frac{w_{H,t}}{A_H\gamma(i)}}_{\substack{\text{High-skill labor}\\ \text{unit cost at task } i}} \qquad \underbrace{\frac{\psi w_{H,t}}{q_t(i)}}_{\substack{\text{Machine}\\ \text{unit cost at task } i}} .$$

The machine cost reflects the ψ units of high-skill labor required to produce one unit of the capital input.

To establish the ordering of technologies, consider first the allocation that would arise if a machine with initial productivity $q_0(i)$ were available at every task. The corresponding machine unit cost is $\psi w_H / q_0(i)$.

Lemma 1 (Task allocation under coercion) *Given the productivity schedules of each input, if an interior equilibrium exists, there are cutoff values $0 < I^* < \bar{I} < 1$ such that, if the initial machine technology $q_0(i)$ is available at every task, the cost-minimizing allocation is*

$$\underbrace{[0, I^*)}_{\textit{Machines}} \qquad \underbrace{(I^*, \bar{I})}_{\textit{Coerced low-skill labor}} \qquad \underbrace{(\bar{I}, 1]}_{\textit{High-skill labor}} . \tag{6}$$

Proof. See Appendix B.4.

The intuition is that machines have a comparative advantage in low-indexed tasks, coerced low-skill labor in intermediate tasks, and high-skill labor in high-indexed tasks. These comparative advantages determine the ordering of inputs across tasks. The exact cutoff points, I^* and $\bar{I}$, are equilibrium objects determined jointly with factor prices and factor-market clearing.

At each boundary, firms are indifferent between the adjacent production technologies. At the machine–coerced-labor boundary,

$$\frac{\psi w_H}{q_0(I^*)} = \frac{w_L}{A_L c(I^*)\gamma(I^*)}, \tag{7}$$

while the boundary between coerced low-skill and high-skill labor satisfies

$$\frac{w_L}{A_L c(\bar{I})\gamma(\bar{I})} = \frac{w_H}{A_H \gamma(\bar{I})}. \tag{8}$$

This allocation distinguishes the scope for mechanization from its actual progress. The stationary frontier I^* describes the eventual extent of mechanization, whereas I_t records the tasks already mechanized at date t. When $I_t < I^*$, profitable opportunities remain, but the required machine technologies have not yet been developed. For $0 < I_t \leq I^*$, the allocation at date t is

$$\underbrace{[0, I_t]}_{\text{Machines}} \qquad \underbrace{(I_t, \bar{I}_t)}_{\text{Coerced low-skill labor}} \qquad \underbrace{(\bar{I}_t, 1]}_{\text{High-skill labor}}, \tag{9}$$

where $\bar{I}_t$ is the contemporaneous boundary between the two labor technologies. It adjusts with factor prices so that their unit costs are equal at the boundary. Section 3.8 explains how innovation expands I_t toward I^*, while $\bar{I}_t$ converges to its stationary value $\bar{I}$.

3.6 Equilibrium Prices and Aggregation

Labor-intensive tasks. Labor-intensive non-mechanized tasks are produced competitively. Hence, task prices are equal to their unit costs:

$$p_t(i) = \begin{cases} \dfrac{w_{L,t}}{A_L c(i)\gamma(i)}, & \text{if } I_t < i \leq \bar{I}_t, \\ \dfrac{w_{H,t}}{A_H \gamma(i)}, & \text{if } i > \bar{I}_t. \end{cases} \tag{10}$$

Notice that the price of a task is higher where the labor assigned to that task is less productive. This follows from the law of one price for each type of labor. The law of one price states that high-skill workers receive the same wage $w_{H,t}$ across tasks, while coerced low-skill labor is rented at the same rate $w_{L,t}$ across tasks in the competitive equilibrium. Otherwise, high-skill workers would not supply their labor to low-paying tasks, while owners of coerced labor would not rent their labor to low-paying tasks. Given these common factor prices, differences in task productivity translate directly into differences in task unit costs and hence task prices. Combining the task prices in (10) with the equal-expenditure condition in (5), labor demand at each task is given by

$$l_t(i) = \frac{Y_t}{w_{L,t}}, \qquad h_t(i) = \frac{Y_t}{w_{H,t}}. \tag{11}$$

Thus, firms demand the same amount of coerced low-skill labor at each task assigned to low-skill labor, and the same amount of high-skill labor at each task assigned to high-skill labor.

By integrating labor demand over the equilibrium task ranges and imposing the market-clearing conditions for total low-skill labor supply L and high-skill labor supply H, including the high-skill labor used in producing the capital input, the equilibrium wages are given by

$$w_{Lt} = [\bar{I}_t - I_t]\frac{Y_t}{L} \quad \text{and} \quad w_{Ht} = [1 - \bar{I}_t + \phi I_t]\frac{Y_t}{H}. \tag{12}$$

The step-by-step derivation from labor-market clearing is given in Appendix B.1.

Capital-intensive tasks. In capital-intensive tasks, firms innovate to have the highest quality of capital input (or machine) and to become the monopolist. When there is an innovation, the innovator improves the quality of capital input in task i by λ, such that the frontier quality becomes $q_{t+\Delta t}(i) = (1+\lambda)q_t(i)$. I assume there are competitive fringe firms that have access to a less productive vintage and can imitate the latest technology at the rate $\phi \in [\frac{1}{1+\lambda}, 1)$; thus, all follower firms have access to a vintage of capital input i with productivity $\phi q_t(i)$ at time t. Then the equilibrium price is pinned down by the limit pricing of the monopolist,

$$p_t(i) = \frac{\psi w_{H,t}}{\phi q_t(i)}, \qquad i \in [0, I_t], \tag{13}$$

where $\phi \in (0,1)$ determines the productivity gap between the frontier technology and the competitive fringe. Using (5) and $y_t(i) = q_t(i)k_t(i)$, demand for the capital input is

$$k_t(i) = \frac{\phi Y_t}{\psi w_{H,t}}, \qquad i \in [0, I_t]. \tag{14}$$

Appendix B.2 derives the capital-input price and quantity from fringe competition and equal task expenditure.

We can now aggregate production across tasks. Using the task allocation in (9), input demands in (11), (14) and the input prices in (12), the equilibrium aggregate output is

$$Y_t = Q_t \left(\frac{L}{\bar{I}_t - I_t}\right)^{\bar{I}_t - I_t} \left(\frac{H}{1 - \bar{I}_t + \phi I_t}\right)^{1 - \bar{I}_t + I_t}, \tag{15}$$

Appendix B.3 provides the derivation from the task aggregator and equilibrium input quantities.

Here, $Q_t \equiv \exp\left(\int_0^{I_t} \ln q_t(i)\, di + \int_{I_t}^{\bar{I}_t} \ln[A_L c(i)\gamma(i)]\, di + \int_{\bar{I}_t}^{1} \ln[A_H \gamma(i)]\, di\right) \left(\frac{\phi}{\psi}\right)^{I_t}$ is the TFP term that is the sum of all input productivities by tasks, scaled by the imitation and marginal cost parameter of capital input. For simplicity, I will refer to it as TFP in the text. The aggregate output is a simple Cobb-Douglas production function of labor inputs and the TFP term Q_t. Since the sum of the exponents equals 1, it exhibits constant returns to scale, and $q_t(i)$ works as labor-augmenting technological change at the aggregate level even though it is capital-augmenting at the task level.

3.7 Stationary Task Allocation

Now, I can find the stationary task allocation. Substituting the equilibrium wage rates from (12) into the cutoff conditions in (7) and (8) gives the following two cost-indifference conditions that characterize the equilibrium cutoff values $(I^*, \bar{I})$:

$$\frac{\bar{I} - I^*}{1 - \bar{I} + \phi I^*} \frac{H}{L} q_0(I^*) = \psi A_L \gamma(I^*) c(I^*), \tag{E1}$$

$$\frac{\bar{I} - I^*}{1 - \bar{I} + \phi I^*} \frac{H}{L} A_H = A_L c(\bar{I}). \tag{E2}$$

Appendix B.5 shows the substitution of equilibrium wages into the two cost-indifference conditions step by step.

Uniqueness under coercion, together with general existence, is established in the proposition below and proved in the Appendix.

Proposition 1 (Existence and Uniqueness of Equilibrium):
If an equilibrium $(\bar{I}, I^*)$ *solving* (E1) *and* (E2) *exists, it is unique. Moreover, an equilibrium exists if*

$$\frac{\psi A_H \gamma(I^*_{\max})}{q_0(I^*_{\max})} \geq 1,$$

where $I^*_{\max}$ *denotes the maximum admissible value of* I^* *such that* $\bar{I}$, $I^* \in [0, 1]$. *For the proof and further discussion, see Appendix.*

3.8 Innovation and Automation Process

I refer to improvements that increase the productivity of existing machine technologies as vertical innovation, and to innovations that replace labor with machines as automation, mechanization, or horizontal innovation. Vertical innovation follows the quality-ladder framework of Aghion and Howitt (1992).

Vertical Innovation. As described above, the firm with the frontier technology is the monopolist in its task and sets the limit price in (13). Its flow profit is therefore,

$$\pi_t(i) = p_t(i) y_t(i) - k_t(i) \psi w_{H,t}.$$

Replacing $y(i)$ from (5) and $k(i)$ from (14), I obtain the equilibrium profit,

$$\pi_t(i) = (1 - \phi) Y_t. \tag{16}$$

The revenue and production-cost calculations are shown in Appendix B.2.

The incumbent's profit is the same across tasks and grows at the same rate as aggregate output. Because each individual task has an infinitesimal effect on Y_t, an incumbent's improvement of its own

task productivity has a negligible effect on aggregate output. Moreover, under the limit-pricing structure, its flow profit does not depend on its own quality. Thus, with costly innovation, incumbents do not undertake vertical R&D; innovation is performed by entrants, as in standard Schumpeterian growth models.

$$r_t V_t(i) - \dot{V}_t(i) = \pi_t - \tau(i) V_t(i), \tag{17}$$

where τ is the creative destruction rate at task i. In particular, τ is the rate at which entrants innovate and replace the incumbent firm. Then, on the BGP, the value of being an incumbent monopolist is as follows:

$$V_t^*(i) = \frac{(1-\phi) Y_t}{r - g + \tau}. \tag{18}$$

There is a unit measure of potential entrants per line. They invest $R^v(x)$ in terms of the final good to generate a flow rate of innovation x, where the cost function $R^v(x)$ is given by

$$R_t^v(x(i)) = \kappa x_t(i)^\alpha Y_t,$$

where $\kappa > 0$ and $\alpha > 1$, so the cost function is convex. The cost of innovation is proportional to the GDP of the economy to capture the idea that innovating is more costly as the economy grows. Then the entrant's innovation problem is defined as follows:

$$\max_x \left\{ x_t(i) V_t(i) - \kappa x_t(i)^\alpha Y_t \right\}.$$

Taking the first-order condition and using the balanced-growth-path value $V_t(i) = V_t^*(i)$ in (18) together with the Euler equation (1) yields the optimal R&D decision as follows:

$$x_t(i) = \left[\frac{(1-\phi)}{(\rho + \tau)\kappa\alpha} \right]^{\frac{1}{\alpha - 1}}.$$

Since there is a unit measure of potential entrants per line, this leads to $\tau(i) = x_t(i)$. Therefore, the total innovation in any line i on the BGP is given by

$$(\tau^*)^{\alpha-1}(\rho + \tau^*) = \frac{(1-\phi)}{\kappa\alpha}. \tag{19}$$

Thus the equilibrium creative destruction term of each line is the same and time independent. Moreover, it is independent of the size of the economy, Y_t, and is constant over time. This is a standard size-independent property of the equilibrium innovation rate as in Klette and Kortum (2004) and Akcigit and Kerr (2018).

On the BGP, since the equilibrium innovation rate is τ^* and a measure I^* of tasks is mechanized, aggregate vertical research expenditure is

$$R_t^{v*} = \int_0^{I^*} \kappa(\tau^*)^\alpha Y_t, \mathrm{d}i = I^* \kappa (\tau^*)^\alpha Y_t. \tag{20}$$

Mechanization. Mechanization replaces labor in tasks for which machine technologies have not yet been developed. A unit measure of innovators undertakes research at the marginal task I_t. For inventors, incentives to mechanize depend on the profits that a new machine can earn, the value of those future profits, and the cost of research.

A newly mechanized task begins with initial machine quality $q_0(I_t)$. As the innovator of a new machine must compete with both low-skill labor and a fringe of competitive firms that imitate their technology, they set a limit price equal to the unit cost of whichever of these has the lower unit cost. At low-indexed tasks, competition from imitating firms determines the price, giving the innovator the same profit share as a vertical innovator. As mechanization reaches tasks where labor is relatively more productive, labor competition restricts the price and reduces the profit share. Specifically, the profit of an innovator that mechanizes a marginal task I_t is given by:

$$\pi_t^m(I_t) = \begin{cases} (1-\phi)Y_t, & \text{if } I_t \leq \tilde{I}_t, \\ \left(1 - \frac{\psi w_{H,t} A_L c(I_t)\gamma(I_t)}{w_{L,t} q_0(I_t)}\right) Y_t, & \text{if } \tilde{I}_t < I_t \leq I^*. \end{cases} \tag{21}$$

Here, $\tilde{I}_t$ is the task at which the price constraint switches from imitating machines to labor. Appendix B.2 derives this switching point and the profit expression.

Let V_t^m denote the value of successfully mechanizing the marginal task, and suppress the task index for simplicity. Its value equation is

$$r_t V_t^m - \dot{V}_t^m = \pi_t^m - \tau_t V_t^m. \tag{22}$$

The required return on the invention equals current profits plus the change in its value, minus the expected loss from creative destruction. Research therefore responds to expected future profits, rather than current profits alone.

Mechanization research has the same convex cost structure as vertical research, but I allow $\kappa^m > \kappa$: developing a machine for a new task may be more costly than improving an existing machine. Innovators choose research effort x_t^m to solve

$$\max_{x_t^m} \left\{ x_t^m V_t^m - \kappa^m (x_t^m)^\alpha Y_t \right\}.$$

With a unit measure of innovators, the equilibrium rate of successful mechanization is $x_t^m = \dot{I}_t$. The first-order condition therefore gives

$$\dot{I}_t = \left(\frac{V_t^m}{\alpha \kappa^m Y_t} \right)^{\frac{1}{\alpha - 1}}. \tag{23}$$

More valuable inventions encourage research and faster mechanization, whereas higher research costs slow it down. Total research expenditure is the sum of vertical and mechanization research expenditure:

$$R_t = R_t^v + R_t^m = \int_0^{I_t} \kappa x_t(i)^\alpha Y_t \, di + \kappa^m (x_t^m)^\alpha Y_t.$$

Thus, the model features a transition path with increasing mechanization and a rising capital

share, consistent with nineteenth-century capital deepening (Abramovitz and David, 1973; Gallman and Rhode, 2022). As I_t approaches I^*, the profit share from mechanizing the marginal task falls toward zero, and mechanization effort vanishes at the stationary frontier. Thus, tasks close to the stationary mechanization frontier may take many years to be mechanized, even when mechanization is profitable. Section 3.11 characterizes how mechanization, vertical innovation, and consumption evolve jointly during this transition. These dynamics also govern the adjustment toward a higher mechanization frontier following emancipation.

On the balanced-growth path, the realized allocation in (9) coincides with (6). No further task can be profitably mechanized, so $x^{m*} = R_t^{m*} = 0$ and total research expenditure consists entirely of vertical research expenditure in (20):

$$R_t^* = R_t^{v*}.$$

3.9 Balanced Growth Path Equilibrium

Along the balanced-growth path, task allocation is stationary at $I_t = I^*$ and $\bar{I}_t = \bar{I}$. Aggregate output is given by

$$Y_t = Q_t \left(\frac{L}{\bar{I} - I^*} \right)^{\bar{I} - I^*} \left(\frac{H}{1 - \bar{I} + \phi I^*} \right)^{1 - \bar{I} + I^*}, \tag{24}$$

where

$$Q_t \equiv \exp\left(\int_0^{I^*} \ln q_t(i)\, di + \int_{I^*}^{\bar{I}} \ln[A_L c(i) \gamma(i)]\, di + \int_{\bar{I}}^{1} \ln[A_H \gamma(i)]\, di \right) \left(\frac{\phi}{\psi} \right)^{I^*}.$$

Since task allocation is fixed, output changes only through improvements in machine quality. The following proposition gives the resulting growth rate.

Proposition 2 (Balanced-Growth Rate): *The balanced-growth rate of aggregate output is*

$$g^* = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{Q}_t}{Q_t} = I^* \tau^* \ln(1 + \lambda). \tag{25}$$

Proof. See Appendix B.6. ■

The growth rate of the economy is determined by the measure of mechanized tasks I^*, the innovation rate τ^* at each of those tasks, and the innovation step size $\ln(1 + \lambda)$. Intuitively, the aggregate growth rate depends on how many tasks are mechanized, at what rate innovation occurs within those tasks, and how much productivity increases with each innovation.

I now define a balanced-growth-path equilibrium.

Definition 1 (Balanced Growth Path Equilibrium) *A balanced growth path equilibrium is an equilibrium allocation in which prices and quantities either grow at constant rates or remain stationary.*

A balanced growth path equilibrium of this economy consists of the tuple

$$\left\{ I^*, \bar{I}, \tau^*, g^*, r^*, w_{L,t}^*, w_{H,t}^*, Y_t^*, C_t^*, R_t^*, Q_t^*, \{p_t^*(i), l^*(i), h^*(i), k^*(i), V_t^*(i)\}_{i \in [0,1]} \right\}_{t \geq 0},$$

such that the following conditions hold:

1. $\{l^*(i), h^*(i), k^*(i), p_t^*(i)\}_{i\in[0,1]}$ *satisfy the task-level input demands and pricing conditions* (11), (14), (10), *and* (13).
2. *The equilibrium wages* $w_{L,t}^*$ *and* $w_{H,t}^*$ *satisfy* (12).
3. *The stationary task allocation is characterized by* $(I^*, \bar{I})$ *satisfying* (E1) *and* (E2), *with the task allocation given by* (6).
4. *Vertical innovation occurs at the constant rate* τ^* *on* $i \in [0, I^*]$, *where* τ^* *satisfies* (19), *and* $V_t^*(i)$ *satisfies* (18).
5. *The mechanization frontier is stationary at* $I_t = I^*$, *so horizontal innovation is zero.*
6. *Aggregate research expenditure* $R_t^* = R_t^{v*}$ *satisfies* (20).
7. *Aggregate output* Y_t^* *and aggregate productivity* Q_t^* *satisfy* (24).
8. *Aggregate consumption* C_t^* *satisfies the resource constraint* (4).
9. *The quantities* Y_t^*, C_t^*, R_t^*, *and* Q_t^*, *the wages* $w_{L,t}^*$ *and* $w_{H,t}^*$, *and incumbent values* $V_t^*(i)$ *grow at the common balanced-growth rate* g^*, *where* g^* *satisfies* (25).
10. *The equilibrium interest rate* r^* *satisfies the household Euler equation* (1).

3.10 Central Theoretical Results

I now summarize the main theoretical implications of the model.

The first result shows how coercion affects the long-run mechanization frontier. Dividing (E1) by (E2) gives the coercion-adjusted set of automatable tasks

$$\frac{q_0(I^*)}{\gamma(I^*)} = \psi A_H \frac{c(I^*)}{c(\bar{I})}. \tag{26}$$

Equation (26) is a key equilibrium condition of the model, as it links the automation threshold I^* directly to the coercion technology. The next proposition states the first key result of the model.

Proposition 3 (Lower Automation Threshold under Coercion): *Let* I^* *denote the equilibrium automation frontier when the coercion technology is available,* $c(i) > 1$, *and let* I^{**} *denote the equilibrium automation frontier when coercion is not available, so that* $c(i) = 1$ *for all* i. *Coercion always leads to a lower automation frontier:*

$$I^* < I^{**}.$$

Proof of Proposition 3. The two equilibrium conditions are $q_0(I^*)/\gamma(I^*) = \psi A_H\, c(I^*)/c(\bar{I})$ and $q_0(I^{**})/\gamma(I^{**}) = \psi A_H$. Because $\bar{I} > I^*$ and c is strictly decreasing, $c(I^*)/c(\bar{I}) > 1$, so the right-hand side is larger under coercion. Since $q_0(i)/\gamma(i)$ is strictly decreasing, this implies $I^* < I^{**}$. ■

Coercion therefore reduces the set of tasks that are eventually mechanized. Thus, there is less mechanization in an economy with coercion than in a free economy. Thus, the model mirrors Stylized Fact 3: the South had a lower capital share than the North.

This effect also has a dynamic implication because vertical innovation can take place only on tasks that have first been profitably mechanized. Tasks that remain outside the mechanization frontier do not

receive subsequent improvements in machine quality. Thus, the effect of coercion on mechanization translates into a lower long-run growth rate. The next proposition formalizes this implication.

Proposition 4 (Lower Balanced-Growth Rate under Coercion): *Let g^* denote the BGP growth rate under coercion and g^{**} the BGP growth rate without coercion. Then, the balanced-growth rate is lower in the coercive economy:*

$$g^* < g^{**}.$$

Proof of Proposition 4. From (25), the balanced-growth rate is determined by the measure of mechanized tasks, the vertical innovation rate, and the innovation step size. The equilibrium vertical innovation rate τ^* and the innovation step size λ are unaffected by the coercion technology and are therefore the same in the two economies. Hence, the difference in long-run growth is entirely determined by the mechanization frontier. Proposition 3 establishes that $I^* < I^{**}$. Therefore, $I^*\tau^* \ln(1+\lambda) < I^{**}\tau^* \ln(1+\lambda)$, which implies $g^* < g^{**}$. ■

This proposition provides the central theoretical result of the paper by establishing a direct link between coercion and lower long-run economic growth. The result connects naturally to the empirical patterns documented above. In particular, it is consistent with the first empirical fact: the income divergence between the U.S. South and North over the nineteenth century. The model provides a mechanism through which the greater reliance on enslaved labor in the South can contribute to this divergence by reducing mechanization and, consequently, long-run productivity growth. The result is also consistent with the subsequent decline in the North–South growth-rate differential observed after 1880, in the post-emancipation period.

Corollary 1 (Lower Aggregate Research Intensity under Coercion) *The aggregate R&D share is lower under coercion than in the free economy.*

Proof. Using $R_t^* = R_t^{v*}$ and (20), and letting R^{**} denote aggregate BGP research expenditure in a free economy, we obtain

$$\frac{R_t^*}{Y_t^*} = I^*\kappa(\tau^*)^\alpha < I^{**}\kappa(\tau^*)^\alpha = \frac{R_t^{**}}{Y_t^{**}}.$$

Since the equilibrium innovation intensity τ^* is unchanged across the two economies and Proposition 3 establishes that $I^* < I^{**}$, the result follows. ■

This corollary maps directly into the empirical patterns documented above. The model predicts lower aggregate research intensity in the coercive economy, consistent with the lower level of patenting observed in the South relative to the North, as documented in Stylized Fact 2. More specifically, the mechanism is closely related to the lower patenting observed before emancipation in tasks that were more intensively performed by enslaved labor. Thus, both the aggregate regional pattern and the task-level evidence are consistent with the model's prediction that coercion reduces the scope for innovation.

Proposition 5 (Mechanization Following the Removal of Coercion):

Suppose coercion is permanently removed at date T, and let I_T denote the mechanization frontier inherited from the coercive economy. Since $I_T \leq I^* < I^{**}$, horizontal innovation remains profitable after the institutional

change, and the mechanization frontier converges toward the free-economy frontier:

$$\lim_{t\to\infty} I_t = I^{**}.$$

Moreover, if the economy is initially at, or sufficiently close to, the stationary coercive frontier I^*, the removal of coercion increases the profitability of horizontal innovation and therefore raises horizontal innovation effort.

Proof of Proposition 5. Under coercion, the realized mechanization frontier satisfies $I_T \leq I^*$. Proposition 3 establishes that $I^* < I^{**}$. Hence,

$$I_T \leq I^* < I^{**}.$$

Following the removal of coercion, the relevant limiting frontier becomes I^{**}. Therefore, as long as $I_t < I^{**}$, mechanizing the marginal task has positive value and horizontal innovation effort is positive. The mechanization frontier consequently expands. As $I_t \to I^{**}$, the profitability of mechanizing the marginal task converges to zero, so horizontal innovation effort vanishes. Hence,

$$\lim_{t\to\infty} I_t = I^{**}.$$

If the economy is initially at, or sufficiently close to, I^*, the profitability of additional mechanization under coercion is small. The removal of coercion raises the relevant limiting frontier from I^* to I^{**} and therefore increases the value of mechanizing the marginal task, which raises horizontal innovation effort. ■

This proposition implies that, following emancipation, tasks in the interval (I^*, I^{**}), which would not have been mechanized under coercion, become profitable to mechanize. More generally, starting from the inherited mechanization frontier I_T, the economy progressively mechanizes tasks in the interval (I_T, I^{**}). If the economy is initially at, or sufficiently close to, the coercive stationary frontier I^*, the removal of coercion generates a substantial increase in horizontal innovation effort and hence in the rate of mechanization. As the frontier advances, progressively higher-indexed tasks are mechanized until the economy approaches I^{**}.

From an empirical perspective, the model therefore predicts that technologies associated with tasks previously performed intensively by enslaved labor should experience a substantial increase in patenting following emancipation relative to the pre-emancipation period. This result maps directly to the empirical finding of increased patenting in slave-replacing technologies following emancipation, shown in Figure 7.

Taken together, the theoretical results show that the model can jointly rationalize the empirical patterns documented above and provide a unified mechanism linking the task-level effects of coercion to long-run economic growth. In particular, the model connects lower mechanization and innovation in slave-intensive tasks to lower aggregate research intensity and slower long-run growth. Dynamically, the model also generates an increase in mechanization and innovation following emancipation, together with a gradual increase in economic growth as the economy transitions toward the free-economy mechanization frontier. In the next section, I quantify the model and assess how well this mechanism can account for the observed empirical patterns. Before doing so, I characterize the

transition dynamics of mechanization and vertical innovation in the next subsection, which will be useful for the quantitative exercise.

3.11 Transition Dynamics

The innovation decisions in Section 3.8 determine the incentives to mechanize new tasks and improve existing machines through vertical innovation. I now characterize how mechanization, vertical innovation, and consumption evolve jointly as the economy approaches its balanced-growth path. The transition is needed for the quantitative exercise and to match the data. The U.S. capital share increased from approximately 14 percent in 1800 to 30 percent in 1900. Thus, the nineteenth century was a period of transition in mechanization and capital deepening. The model therefore needs to capture the transition dynamics of mechanization and vertical innovation, as well as the resulting changes in consumption and the increase in the capital share. I first derive the laws of motion for the mechanization frontier, the relative value of mechanization, and the vertical innovation rate. I then characterize how these laws of motion interact with the resource constraint to determine the evolution of consumption.

I suppress task indices and retain V_t and V_t^m for the values of successful vertical and mechanization innovations. The vertical research first-order condition, evaluated at $x_t = \tau_t$, gives

$$V_t = \alpha\kappa\tau_t^{\alpha-1}Y_t. \tag{27}$$

Define the relative value of horizontal to vertical innovation as

$$v_t \equiv \frac{V_t^m}{V_t}. \tag{28}$$

Combining equations (27) and (23) yields the law of motion for the mechanization frontier:

$$\dot{I}_t = \tau_t \left(\frac{\kappa}{\kappa^m} v_t\right)^{\frac{1}{\alpha-1}}. \tag{29}$$

Thus, mechanization proceeds faster when the value of opening a new mechanized task is higher relative to the value of improving an existing one (a higher v_t), when mechanization is less costly relative to vertical innovation (a higher κ/κ^m), and when the vertical innovation rate τ_t is higher.

On the balanced-growth path, the marginal mechanization innovation earns zero profits, $\pi^m = 0$. Since its value is stationary, $\dot{V}^m = 0$, the horizontal HJB implies $V^m = 0$, and therefore $v_t = 0$. Equation (29) consequently implies $\dot{I}_t = 0$, which is consistent with the definition of the balanced-growth path. I next derive the law of motion for v_t.

The relative value of mechanization. The value equations (17) and (22) describe the returns to the two types of innovation. Differentiating (28) and using these value equations gives

$$\dot{v}_t = \frac{\pi_t}{V_t}\left(v_t - \frac{\pi_t^m}{\pi_t}\right).$$

Since equilibrium vertical profits satisfy $\frac{\pi_t}{Y_t} = (1-\phi)$, equation (27) implies

$$\dot{v}_t = \frac{1-\phi}{\alpha\kappa\tau_t^{\alpha-1}}\left(v_t - \frac{\pi_t^m}{\pi_t}\right). \tag{30}$$

Equation (30) distinguishes current profitability from forward-looking incentives. The ratio π_t^m/π_t measures the current profitability of expanding the mechanization frontier relative to improving an existing mechanized task, whereas v_t incorporates the entire future stream of profits associated with the two types of innovation. At the balanced-growth path, $\pi_t^m = 0$ and $v_t = 0$, so that $\dot{v}_t = 0$, but off the balanced-growth path, $\pi_t^m > 0$, $v_t > 0$, and $\dot{v}_t > 0$ for some tasks.

Resource feasibility. Finally, I characterize how changes in R&D spending along the transition affect the consumption share and consumption growth, which in turn affect the vertical innovation rate τ_t. Recall that the resource constraint, expressed in terms of vertical and horizontal R&D expenditures, is

$$Y_t = C_t + \kappa I_t \tau_t^{\alpha} Y_t + \kappa^m \dot{I}_t^{\alpha} Y_t,$$

where the second term is expenditure on vertical innovation over the measure I_t of mechanized tasks and the third term is expenditure on horizontal innovation. Equivalently,

$$\frac{C_t}{Y_t} = 1 - \kappa I_t \tau_t^{\alpha} - \kappa^m \dot{I}_t^{\alpha}.$$

Taking logs of equation (27) and differentiating with respect to time, and then combining the resulting expression with the Euler equation (1) and the vertical-innovation HJB, gives, after rearranging,

$$(\alpha-1)\frac{\dot{\tau}_t}{\tau_t} = \rho + \tau_t + \frac{d}{dt}\ln\left(\frac{C_t}{Y_t}\right) - \frac{1-\phi}{\alpha\kappa\tau_t^{\alpha-1}}. \tag{31}$$

Together, the state variables (I_t, v_t, τ_t), whose dynamics are given by equations (29), (30), and (31), together with the static task-allocation conditions, characterize the transition outside the balanced-growth path. The economy converges to

$$(I_t, \tau_t, v_t) \longrightarrow \begin{cases} (I^*, \tau^*, 0), & \text{under coercion, } c(i) > 1, \\ (I^{**}, \tau^*, 0), & \text{under no coercion, } c(i) = 1. \end{cases}$$

Thus, the long-run vertical innovation intensity is τ^* in either regime, while the mechanization frontier converges to the lower level I^* under coercion and to I^{**} in the absence of coercion as I have shown in the previous sections.

4 A Quantitative Exercise: the US North and South

4.1 Two Region Extension

To quantify how coercion and its effects on innovation contributed to the economic divergence between the North and South, I extend the model to two regions, indexed by $f \in \{N, S\}$. Before abolition, the South has access to the coercion technology, $c^S(i) = c(i) > 1$, whereas the North does not, $c^N(i) = 1$. Production technologies, innovation technologies, and household preferences otherwise follow the one-region model. Each region produces a differentiated good. Labor is immobile across regions, and asset markets are regionally segmented. The two regions are connected through demand for their goods, while wages, returns on savings, innovation paths, and therefore growth rates can differ across regions.

Regional markets and households. Each region has a representative household that consumes the national final good and supplies high- and low-skill labor, H^f and L^f. Labor is immobile across regions, and households own and save only in firms located in their own region. Consequently, regional wages and rates of return need not be equalized. The Euler equation for each region is

$$\frac{\dot{C}_t^f}{C_t^f} = r_t^f - \rho, \qquad f \in \{N, S\}.$$

Appendix D.1 shows the household problem.

Regional production and demand. Each regional good, Y_t^f, is produced by combining a continuum of region-specific tasks using the same Cobb-Douglas technology as in Section 3.3. The two regional goods are combined competitively into the national final good according to

$$Y_t = \left[\chi^{\frac{1}{\varepsilon}} \left(Y_t^N\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\chi)^{\frac{1}{\varepsilon}} \left(Y_t^S\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \varepsilon > 1, \tag{32}$$

Here, ε is the elasticity of substitution between regional goods, and $\chi \in (0,1)$ governs relative demand. I take the national final-good price as the numeraire and denote regional prices by P_t^f. Cost minimization by national final-good producers implies

$$\frac{P_t^S}{P_t^N} = \left(\frac{1-\chi}{\chi}\right)^{\frac{1}{\varepsilon}} \left(\frac{Y_t^N}{Y_t^S}\right)^{\frac{1}{\varepsilon}}.$$

Define regional income, or GDP, as

$$E_t^f \equiv P_t^f Y_t^f.$$

Thus, Y_t^f measures the quantity of regional output, whereas E_t^f measures its value in units of the national final good. The CES demand system gives the relative value of regional output as

$$\frac{E_t^S}{E_t^N} = \left(\frac{1-\chi}{\chi}\right)^{\frac{1}{\varepsilon}} \left(\frac{Y_t^S}{Y_t^N}\right)^{\frac{\varepsilon-1}{\varepsilon}}.$$

Regional income therefore depends on both the regional production and the regional price.

Regional innovation and task allocation. Within each region, the Cobb–Douglas task aggregator again implies equal expenditure across tasks:

$$p_t^f(i) y_t^f(i) = E_t^f.$$

The wage and profit expressions consequently take the same form as in the one-region model, with regional income replacing the revenue term Y_t:

$$w_{L,t}^f = (\bar{I}_t^f - I_t^f)\frac{E_t^f}{L^f}, \qquad w_{H,t}^f = (1 - \bar{I}_t^f + \phi I_t^f)\frac{E_t^f}{H^f}, \qquad \pi_t^f = (1-\phi)E_t^f.$$

Innovation also follows the one-region model: a unit measure of entrants undertakes vertical research in each mechanized task, while mechanization research takes place at the regional frontier I_t^f. Research costs are scaled by E_t^f rather than Y_t. For example, the cost of vertical research effort x is $\kappa x^\alpha E_t^f$. Thus both returns and costs are scaled with the regional income. This common scale cancels from the normalized research conditions as in the one region model. Similarly, regional income cancels from the wage ratio that determines task allocation.

Segmented labor and asset markets allow these factor prices and innovation decisions to adjust within each region. The CES demand system only determines regional prices, income shares, and national aggregation, while the normalized task-allocation and innovation conditions are the same as in the one-region model. The regional counterparts of (E1) and (E2) are derived in Appendix D.1.

Regional resource constraints. Regional income finances household consumption and research undertaken in that region. Measured in units of the national final good,

$$E_t^f = C_t^f + R_t^{v,f} + R_t^{m,f}, \qquad f \in \{N, S\},$$

where $R_t^{v,f}$ and $R_t^{m,f}$ denote vertical and mechanization research expenditures. With competitive national final-good production, the corresponding national expenditure identity is

$$Y_t = E_t^N + E_t^S.$$

The extension therefore preserves the mechanism linking coercion to mechanization and growth while connecting regional production through a common goods market. I next characterize the regional growth paths and how their differences translate into relative prices and income.

4.2 Balanced Growth Path Equilibrium and Transition Dynamics

Regional growth and relative prices. The one-region results apply to each region accordingly such that North has free stationary values and South has coercive stationary values. Common innovation parameters imply the same stationary vertical innovation intensity τ^* for both regions, determined by (19). Before abolition, the North converges toward the free-labor mechanization frontier I^{**}, while the South converges toward the coercive frontier $I^* < I^{**}$. Conditional on both regions having reached their regime-specific stationary task allocations, (25) gives

$$g_N^* = I^{**}\tau^* \ln(1+\lambda), \qquad g_S^* = I^*\tau^* \ln(1+\lambda).$$

Coercion thus lowers Southern growth by limiting the set of tasks that receive subsequent improvements in machine quality.

Differences in production growth also change relative regional prices. Let $g_{f,t} \equiv \dot{Y}_t^f / Y_t^f$ denote the growth rate of regional output quantities. The CES demand system implies

$$\frac{d}{dt} \ln \left(\frac{P_t^S}{P_t^N} \right) = \frac{1}{\varepsilon}(g_{N,t} - g_{S,t}),$$

and hence

$$\frac{d}{dt} \ln \left(\frac{E_t^S}{E_t^N} \right) = \frac{\varepsilon - 1}{\varepsilon}(g_{S,t} - g_{N,t}).$$

When the North grows faster, Southern output becomes relatively scarce, and therefore its relative price rises. This price response alleviates the decline in the South's relative income as its output quantity and the value of that output do not fall at the same rate relative to the North. The CES block therefore matters for nominal regional income and final good aggregation at the national level even though it leaves the normalized within-region innovation conditions unchanged.

Regional transitions and emancipation. The regions need not initially be at their stationary allocations. Their adjustment depends on inherited mechanization and the incentives to invent new machines and improve existing ones. Scaling profits and research costs by regional income gives the same transition conditions as in Section 3.11. Define $v_t^f \equiv V_t^{m,f} / V_t^f$ as the value of mechanization relative to vertical innovation. The regional transition is characterized by the joint evolution of (I_t^f, τ_t^f, v_t^f):

$$\begin{cases} \dot{I}_t^f = \tau_t^f \left(\dfrac{\kappa}{\kappa^m} v_t^f \right)^{\frac{1}{\alpha-1}}, \\ \dot{v}_t^f = \dfrac{1-\phi}{\alpha\kappa(\tau_t^f)^{\alpha-1}} \left(v_t^f - \dfrac{\pi_t^{m,f}}{\pi_t^f} \right), \\ (\alpha-1)\dfrac{\dot{\tau}_t^f}{\tau_t^f} = \rho + \tau_t^f + \dfrac{d}{dt} \ln \left(\dfrac{C_t^f}{E_t^f} \right) - \dfrac{1-\phi}{\alpha\kappa(\tau_t^f)^{\alpha-1}}, \\ \dfrac{C_t^f}{E_t^f} = 1 - \kappa I_t^f (\tau_t^f)^\alpha - \kappa^m (\dot{I}_t^f)^\alpha, \end{cases} \qquad f \in \{N, S\}. \tag{33}$$

As in the one-region model, mechanization advances when opening a new task is valuable relative to improving an existing machine. The evolution of this relative value reflects the deviation of the value itself from the profitability of the marginal mechanization opportunity, $\pi_t^{m,f} / \pi_t^f$, while the vertical innovation rate responds to innovation profitability and changes in the consumption share. The resource constraint determines the consumption share C_t^f / E_t^f, which is one minus total research expenses as a share of regional income. Together with regional task allocation, these conditions determine each region's path toward its stationary frontier. The derivation is provided in the Online Appendix.

These transition equations are important for describing the effect of the abolition of slavery on the Southern economy. This takes place as follows. Emancipation removes the Southern coercion technology, setting $c^S(i) = 1$ and changing its stationary mechanization target from I^* to I^{**}. Immediately after emancipation, the South starts with the mechanization frontier it inherited. The removal of coercion changes innovation incentives before all newly profitable tasks have been mechanized. Thus these transition equations describe this subsequent adjustment, and the CES demand system translates the resulting regional production paths into relative prices and income.

4.3 Calibration and Estimation

4.3.1 Functional forms

For the quantitative analysis, I parameterize task productivity, the coercion technology, and baseline machine productivity as

$$\gamma(i) = \exp(\beta i), \qquad c(i) = C \exp(-\beta_c i), \qquad q_0(i) = \frac{\bar{q} \exp(-\beta_q i)}{i}. \tag{34}$$

It is convenient to define

$$a \equiv \beta + \beta_q.$$

These functional forms preserve the monotonicity assumptions used in the theoretical analysis while allowing the stationary cutoff conditions to be expressed in terms of a small number of parameters.

4.3.2 Externally Calibrated Parameters

I set the discount rate parameter, ρ, to 2 percent, which is approximately equivalent to an annual discount factor of 97 percent. The imitation parameter, ϕ, is set to 0.9 to match the 10 percent profitability rate that Vedder and Gallaway (1980) estimated for 1860 census manufacturing firms. Since $\phi \geq \frac{1}{1+\lambda}$, I choose λ to satisfy $\phi = \frac{1}{1+\lambda}$. This implies an innovation step size of 0.11. The worker ratio in the South is matched to the ratio of enslaved to free workers before abolition as $(L/H)_{pre} = 1.04$ from the 1860 census data, and to $(L/H)_{post} = 0.707$ following the labor supply decline of 32 percent among the formerly enslaved population after emancipation as estimated by Ransom and Sutch (2001). High-skill worker productivity, A_H, is normalized to 1, and low-skill worker productivity is set to 0.43 using the daily wage ratio between unskilled and skilled daily workers in the South Atlantic in 1870 from the dataset provided by Margo (2002).

Because this is a historical setting, direct evidence on R&D expenditures or other measures of in-

novation investment is not available. Innovation in this period was also done differently from today. Many innovations were developed by individual inventors rather than by firms with R&D labs. Nevertheless, innovation still required costly inputs, such as investing in equipment and materials, as well as the inventor's time and the associated opportunity cost. Thus, although R&D expenditure in the modern sense is not directly observable in this setting, innovative activity still required costly inputs. I summarize these inputs through the innovation cost function in the model, following the standard lab-equipment approach to innovation. Many inventors of that time commercialized their ideas and innovations as entrepreneurs, as illustrated by figures such as Eli Whitney, Cyrus McCormick, and John Deere.

In the absence of direct historical evidence on the curvature, α, of these costs, I discipline this parameter using the empirical literature on R&D and innovative output. In the literature, there are estimates of the elasticity of innovation output, often measured by patents, with respect to R&D expenditures. This literature finds values for this elasticity around 0.5 (Griliches, 1990; Hall and Ziedonis, 2001; Blundell et al., 2002). [15] This elasticity of approximately 0.5 corresponds to a quadratic innovation cost function in the model. Therefore, I set $\alpha = 2$.

I set the elasticity of substitution between Northern and Southern goods to $\varepsilon = 5$ in the benchmark specification and report robustness to alternative values. This value is consistent with the quantitative trade literature. Anderson and van Wincoop (2003) use an elasticity of 5 as a benchmark in their structural-gravity analysis, drawing on estimates by Hummels (1999) that imply elasticities of approximately 4.8, 5.6, and 6.9 at progressively finer levels of industry aggregation. More recent work on the economic geography of U.S. slavery by Allen et al. (2025) uses across-location elasticities of 9.22 in agriculture, 4.05 in manufacturing, and 5.98 in services. Since the regional goods in this paper are broad composite regional goods, including both agricultural and manufacturing production, I take $\varepsilon = 5$ as the benchmark value, but I report robustness in Appendix E.3. In short, the model is not very sensitive to this parameter.

The regional substitution elasticity also has an important quantitative implication. The historical data exhibit a substantial change in the nominal income ratio between the South and the North (see Figure 1). With a low elasticity of substitution, a decline in Southern relative real output generates a relatively large offsetting increase in the relative price of Southern goods. As a result, a given change in relative real output produces a smaller change in relative nominal income.[16]Thus, the lower is ε, the larger the required divergence in North–South real output and productivity needed to reproduce the observed change in the nominal income ratio. For example, with $\varepsilon = 5$, a 1 percent change in relative real output translates into a 0.8 percent change in relative nominal income. Low substitution elasticities (close to unity) therefore require a larger divergence in Northern and Southern productivity

[15] I interpret these estimates as informative about the curvature of the relationship between the innovation cost and innovation output, rather than about the level of innovation costs in the historical period. In the model, time variation in the scale of innovation costs is captured by aggregate output, Y_t, while κ governs the overall level of innovation costs. I therefore treat the curvature parameter α as a more general feature of the innovation technology that can be disciplined using evidence from other periods.

[16] With a constant CES distribution parameter,

$$\Delta \ln \left(\frac{E_S}{E_N} \right) = \frac{\varepsilon - 1}{\varepsilon} \Delta \ln \left(\frac{Y_S}{Y_N} \right).$$

to reproduce the observed regional income divergence. [17]

The main qualitative result is robust across the elasticity grid: the coercive mechanization frontier remains below the free-labor frontier. For all tested values with $\varepsilon \geq 4$, each of the nine moments is within 3.6 percent of its target, and long-run Northern growth remains close to 1.23 percent per year. The same frontier ordering holds at lower elasticities, although the fit deteriorates substantially (Appendix Table 17).

The robustness exercise also provides information about the conditions ensuring uniqueness of the stationary equilibrium. A sufficient condition for global monotonicity of the stationary equilibrium condition is

$$a + 1 - \beta_c > 0, \qquad a \equiv \beta + \beta_q.$$

This condition holds for all reported values with $\varepsilon \geq 5$, but fails at the lower elasticities in the grid. Its failure, however, does not imply that an equilibrium does not exist or that it is necessarily multiple. Numerical checks find one interior stationary root in each regime at all reported elasticities, although this does not establish global uniqueness when the sufficient condition fails. I report these results in Appendix E.3.

I report the externally calibrated set of parameters in the table 1

Table 1: Externally Calibrated Parameters

Parameter	Value	Description
α	2	Innovation elasticity
ε	5	Elasticity of substitution across regional goods
ρ	0.02	Discount rate
ϕ	0.90	Imitation parameter $= 1 -$ profit rate
λ	1/9	Step size of vertical innovation
A_H	1	High-skill productivity normalization
A_L	0.43	Low-skill productivity
$(L^S/H^S)_{pre}$	1.04	Prewar labor ratio
$(L^S/H^S)_{post}$	0.7072	Postwar labor ratio

4.3.3 Quantitative Implementation

I compute the transition for a given parameter vector

$$\Theta = (I^{**}, I^*, \tau^*, \xi, I_{N,1800}, I_{S,1800}, \bar{I}, \beta_c), \qquad \xi \equiv \frac{\kappa^m}{\kappa}. \tag{35}$$

For a given parameter vector Θ in (35), I compute the transition implied by the equilibrium conditions derived above. I take the South–North GDP-per-worker ratio in 1800 from the data and use it to normalize the initial relative income level. From that point onward, relative GDP per worker is gener-

[17] The poorest overall fit occurs at the lowest value considered, $\varepsilon = 2$. At this value, the estimated coercive mechanization frontier is about 34 percent of the free-labor frontier, $I^*/I^{**} \simeq 0.34$. Since regional balanced-growth rates are proportional to their respective stationary mechanization frontiers, Northern productivity growth is almost three times Southern productivity growth under the pre-abolition institutional regimes. The model implies a long-run Northern growth rate of approximately 1.64 percent per worker per year, compared with the target of 1.23 percent. At the benchmark $\varepsilon = 5$, the implied Northern growth rate is approximately 1.233 percent.

ated by the model. Using GDP per worker is particularly useful because it abstracts from population size and focuses on productivity per worker. Thus, it allows the model to focus on productivity as the main measure. It also allows the model to avoid taking a position on population growth to match aggregate GDP, as the target is GDP per worker.[18] GDP per worker in both regions is used to construct the change in regional output share, as no assumption is made about population growth. The CES demand system then maps regional output into regional expenditures, which are used to construct the national moments.

The estimation proceeds in five steps. Numerical details are reported in Appendix E.1.

1. **Choose candidate equilibrium values and initial conditions.** The optimizer proposes a candidate vector Θ, defined in (35). This includes the free and coercive mechanization frontiers, stationary innovation intensity, and Northern and Southern mechanization in 1800.
2. **Recover the remaining structural parameters.** The stationary cutoff conditions (E1)–(E2) and the innovation condition (19) are used to recover the productivity schedules and innovation costs consistent with the candidate equilibrium values.
3. **Compute the equilibrium transition paths.** Solve the transition system (33) to construct the paths that converge to the stationary equilibria under coercion and free labor.
4. **Simulate the historical economy.** Northern and Southern mechanization are initialized at their candidate 1800 values. The North follows the free-labor path throughout the simulation period, while the South switches to the free-labor path at emancipation in 1865. In addition, the postwar change in Southern relative labor supply is imposed at the same date.
5. **Match the historical moments.** The simulated moments are compared with their empirical counterparts. The optimizer updates the candidate parameter vector to minimize the sum of squared normalized differences for all nine moments, as defined in (36).

Estimation I estimate the parameters in (35) by nonlinear minimum distance. Let $\text{data}(j)$ denote empirical moment j and let $\text{model}(j;\Theta)$ denote its model counterpart. Because the targeted moments have different units and magnitudes, I normalize each difference by the average absolute magnitude of its empirical and model counterparts, as is standard in the literature, to prevent moments from mechanically receiving different weights due to their scale or units. The estimator solves

$$\widehat{\Theta} = \arg\min_{\Theta\in\mathcal{P}} \sum_{j=1}^{9} \left[\frac{\text{model}(j;\Theta) - \text{data}(j)}{\frac{1}{2}\left|\text{model}(j;\Theta)\right| + \frac{1}{2}\left|\text{data}(j)\right|} \right]^2, \tag{36}$$

[18] GDP per worker follows directly from aggregate output in (15). For example, dividing aggregate output by the total number of workers $N = L + H$ gives

$$\frac{Y_t}{N} = Q_t \left(\frac{L/N}{\bar{I} - I^*}\right)^{\bar{I}-I^*} \left(\frac{H/N}{1 - \bar{I} + \phi I^*}\right)^{1-\bar{I}+I^*}.$$

Hence, what matters for GDP per worker is the share of each type of labor, L/N and H/N, rather than the absolute levels. Along the transition path, the corresponding expression is evaluated at the time-varying mechanization rate I_t and labor allocation margin $\bar{I}_t$. Changes in GDP per worker therefore reflect both changes in productivity, Q_t, and changes in task allocation associated with I_t and, under coercion, $\bar{I}_t$. Within each institutional regime, relative labor supply is held fixed throughout the simulation, except for the postwar change in Southern labor supply described below.

subject to the parameter restrictions implied by the model.

I construct estimates of the national capital income share using historical capital-stock estimates and GNP series in (Gallman and Rhode, 2022), together with New England bond yields (Homer and Sylla, 2005).[19]

The South–North GDP-per-worker moment is constructed using historical regional income estimates (Lindert and Williamson, 2012; Klein, 2013) and regional worker counts (Weiss, 1992, 2006; United States. Bureau of the Census, 1914). Given the benchmark value of ε, the CES demand system is used to convert nominal regional income into relative real output, which is then expressed in per-worker terms. The South–North GDP-per-worker level in 1800 is used only as a normalization; the change between 1800 and 1860 is the moment targeted in estimation.

The late-antebellum free-to-coerced labor hire-cost ratio is taken from Rönnbäck (2021).[20] The productivity ratio between slave plantations and free farms is based on the estimate of Fogel and Engerman (1977), consistent with the plantation-productivity evidence discussed above. Finally, I target a long-run Northern GDP-per-worker growth rate of 1.23 percent per year, computed as the annualized nineteenth-century growth rate from the GDP series of Johnston and Williamson (2026).

Table 2 reports the targeted data moments and their corresponding model estimates.

Table 2: Targeted Moments

Moment	Data	Model
National capital income share, 1800	0.1448	0.1450
National capital income share, 1860	0.2511	0.2465
National capital income share, 1890	0.2834	0.2866
National capital income share, 1900	0.2956	0.2975
Change in log S/N GDP per worker, 1800–1860	-0.3473	-0.3465
Long-run Northern GDP/worker growth	0.0123	0.0123
High-skill free / low-skill coerced hire-cost ratio	1.6900	1.6933
Coercive/free plantation productivity ratio	1.3500	1.3547
North/South reproducible capital-stock ratio, 1800	1.5650	1.5662

The moments jointly discipline the main features of the transition and the equilibrium. The capital-share observations discipline the initial positions of the two regions, the speed of mechanization, and the free-labor frontier I^{**}. The prewar relative-growth moment is particularly informative about the

[19] In the model, capital expenditure in region f is the expenditure on machine inputs across mechanized tasks. Thus, the corresponding object in the model is the output share of expenditure paid for capital services. Since one unit of capital costs $\psi w_{H,t}^{f}$, regional capital expenditure is

$$K_t^{\exp,f} = \int_0^{I_t^f} \psi w_{H,t}^{f} k_t^{f}(i)\, di = \phi I_t^{f} E_t^{f}.$$

I use this expenditure share as a proxy for the regional capital income share. Using the weights given by each region's GDP share, I obtain the total share of capital expenditure in the national economy:

$$s_{K,t}^{M} = \frac{\phi I_t^{N} E_t^{N} + \phi I_t^{S} E_t^{S}}{E_t^{N} + E_t^{S}} = \phi \left[\theta_t^{N} I_t^{N} + \theta_t^{S} I_t^{S}\right], \qquad \theta_t^{f} \equiv \frac{E_t^{f}}{E_t^{N} + E_t^{S}}.$$

Thus, regional mechanization determines the regional capital share, while regional expenditures determine the weights used to construct its national counterpart.

[20] The hiring-cost premium I use from Rönnbäck (2021) is based on the underlying annual slave-hiring rates for the Upper South reported by Evans (1962).

gap between the coercive and free-labor frontiers, I^*/I^{**}. The hire-cost moment primarily disciplines the allocation of manual tasks and hence $\bar{I}$, while the plantation-productivity moment disciplines the coercion schedule $c(i)$. Finally, long-run Northern growth is particularly informative about the stationary innovation intensity τ^*.

The resulting parameter estimates are reported in Table 3. They correspond to the joint solution of the minimum-distance problem in (36), using all nine targeted moments simultaneously.

Table 3: Estimated Parameters

Parameter	Description	Estimate
I^{**}	Free-labor stationary mechanization frontier	0.3576
I^*	Coercive stationary mechanization frontier	0.1632
τ^*	Stationary vertical innovation intensity	0.3272
$\xi = \kappa^m/\kappa$	Relative cost of horizontal innovation	97.40
$I_{N,1800}$	Initial Northern mechanization frontier	0.1937
$I_{S,1800}$	Initial Southern mechanization frontier	0.1275
$\bar{I}$	Stationary coercive labor-allocation cutoff	0.5375
β_c	Slope of the coercion schedule	3.4858

Conditional on the estimated parameter vector and the externally calibrated parameters, the stationary equilibrium conditions imply the remaining technology and innovation parameters.[21] These recovered parameters are then used to construct the free and coercive transition paths for I_t, τ_t, and v_t. Table 4 reports their values.

Table 4: Implied Parameters

Parameter	Description	Value
C	Level of the coercion schedule	8.94
$a = \beta + \beta_q$	Relative slope of machine and labor productivity	2.68
$\psi/\bar{q}$	Relative machine-input cost	1.073
κ	Vertical innovation cost parameter	0.440
κ^m	Horizontal innovation cost parameter	42.87

4.4 Model Fit and Implied Transition Dynamics

Using the estimated parameter vector $\widehat{\Theta}$, together with the implied and externally calibrated parameters, I simulate the regional transition paths from 1800 onward. The resulting paths are

$$\left\{I_t^N, \tau_t^N, v_t^N\right\} \qquad \text{and} \qquad \left\{I_t^S, \tau_t^S, v_t^S\right\}.$$

The North follows the free-labor transition throughout the sample. The South instead follows the coercive transition until abolition in 1865, after which it switches to the free-labor transition from its

[21] Intuitively, I invert the stationary equilibrium conditions to recover the underlying parameters that rationalize the estimated equilibrium objects. Given I^*, I^{**}, and $\bar{I}$, the stationary cutoff conditions determine the parameters governing the slopes and relative levels of the productivity schedules consistent with these equilibrium frontiers. Similarly, given τ^*, the stationary innovation condition determines κ, while κ^m follows from the estimated ratio $\xi = \kappa^m/\kappa$.

inherited level of mechanization.

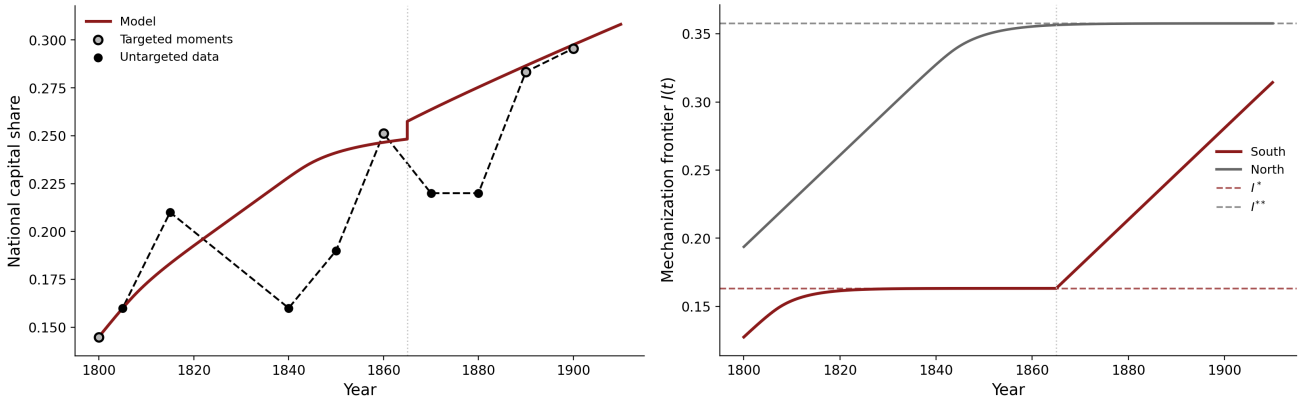
The 1800 South–North GDP-per-worker ratio provides the initial normalization. Subsequent relative GDP-per-worker dynamics are generated by the model's transition equations. Along the transition, vertical innovation raises productivity, while horizontal innovation expands the set of mechanized tasks and changes the static allocation of labor across tasks.

At each date, the model also implies a national capital share. Regional capital expenditure shares are aggregated using the model-implied nominal expenditure shares,

$$s_{K,t}^{M} = \phi \left[\theta_t^N I_t^N + \theta_t^S I_t^S \right], \qquad \theta_t^f \equiv \frac{E_t^f}{E_t^N + E_t^S}.$$

The national capital share depends on both regional mechanization rates and the regions' nominal expenditure weights, and changes over time as these evolve.

Figure 9a compares the model-implied national capital share with the historical capital-income-share series. The model closely matches the targeted observations in 1800, 1860, 1890, and 1900, which discipline the long-run rise in capital intensity over the nineteenth century. The associated mechanization paths are shown in Figure 9b. The mechanism behind this increase is as follows. Before abolition, the North operates under free labor and converges toward the free-labor stationary mechanization frontier I^{**}. On the other hand, the South operates under coercion and converges toward the substantially lower frontier I^{*}. Because coerced labor has lower unit cost in the tasks in which coercion is most effective, mechanizing additional tasks becomes unprofitable at a lower frontier. As the South approaches I^{*}, the incentive to undertake horizontal R&D therefore disappears. Once abolition occurs in 1865, a new range of tasks between I^{*} and I^{**} becomes profitable to mechanize in the South. Inventors, therefore, direct horizontal R&D toward these tasks and introduce new machines, contributing to the rise in the mechanization rate in the South and the national capital share after abolition.



(a) National Capital Expenditure Share

(b) Mechanization rate I_t

Figure 9

The model reproduces the long-run rise in the capital share, but not the declines around 1870 and 1880, which are not targeted in the estimation. This is mainly because once a task is mechanized in

the model, it remains mechanized. Since the capital share is directly determined by the fraction of mechanized tasks, it increases as the economy approaches its stationary mechanization frontier. The model also does not feature wartime destruction or other shocks that may temporarily affect measured capital stock or capital income. This is particularly relevant for post-Civil War observations, as capital destruction in the South is well documented in the literature (Goldin and Lewis, 1975; Feigenbaum et al., 2022). The relatively low observations in 1840 and 1850 may also reflect changes in the computed rate of return or the construction of the historical capital-income-share series. Thus, the quantitative exercise mainly captures the secular rise in the capital share over the nineteenth century, rather than short-run fluctuations in the capital share.

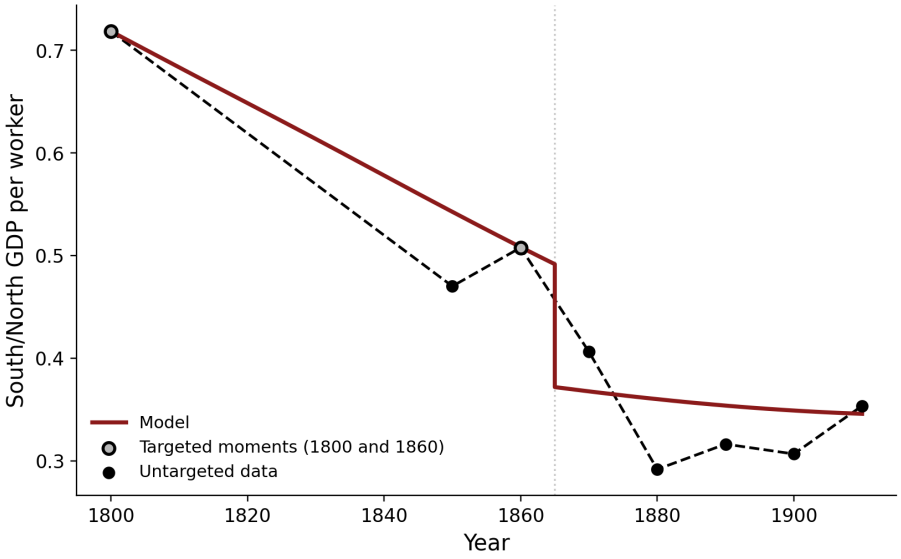


Figure 10: South–North GDP per Worker: Model and Data

Next, I show how well the model captures the South–North GDP-per-worker ratio from 1800 to 1910. The model-generated ratio is normalized to equal the observed value in 1800, while the estimation targets only the pre-abolition growth rate differential between the two regions from 1800 to 1860. Apart from the 1800 normalization and the targeted 1800–1860 change, all subsequent observations are untargeted. The estimated model traces these data points quite closely and captures the long-run evolution of the growth differential between the two regions reasonably well.

The time path in Figure 10 can be divided into three main periods. First, before abolition, the two economies follow different transition paths generated by the sequences of state variables governing mechanization, vertical innovation, and horizontal innovation, given by the region-specific tuples $\{I_t^N, \tau_t^N, v_t^N\}$ and $\{I_t^S, \tau_t^S, v_t^S\}$. The North follows the free-labor transition toward the mechanization frontier I^{**}, whereas the South follows the coercive transition toward the lower frontier I^*. The lower mechanization rate in the South limits the set of tasks over which vertical innovation takes place and therefore generates slower productivity growth relative to the North. As a result, the South–North GDP-per-worker ratio declines steadily and sharply before abolition.

The second period begins with abolition in 1865 and the end of the Civil War. Once slavery is

abolished, the labor-coercion technology is no longer available to Southern firms; thus, the model shuts down this channel by letting $c(i) = 1$ for every task i. This generates an immediate decline in the productivity advantage previously associated with coercion. At the same time, the quantitative model imposes a decline in the effective low- to high-skill labor ratio from 1.04 to 0.707, capturing the decline in the labor supply of formerly enslaved workers. This change partly offsets the direct productivity loss from the removal of coercion.

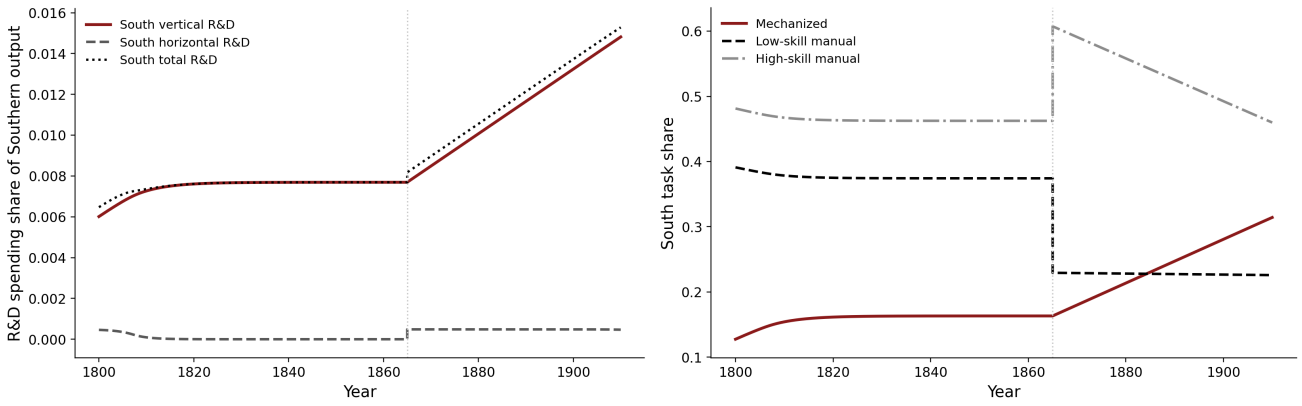
The intuition is the following. As the effective supply of low-skill labor falls, its relative scarcity increases, and some manual tasks are reallocated toward high-skill labor. Since mechanization itself adjusts gradually rather than jumping immediately, this labor reallocation absorbs part of the short-run adjustment. Because high-skill labor has higher baseline productivity, $A_H > A_L$, the reallocation partly mitigates the productivity decline associated with the removal of coercion. The corresponding change in the model's task allocation is shown in Figure 11b, where the share of tasks performed directly by high-skill labor jumps upward immediately after abolition. Thus, the overall effects of the loss of the coercion technology, the change in relative labor supply, and the general-equilibrium effect through changes in wages are captured by the vertical decline in the figure 10 at 1865 following abolition.

As the model does not include any other negative effects of the Civil War and Reconstruction period on the Southern economy, as in the case of the capital-share graph, it does not fully capture the sharp decline in the Southern economy from 1860 to the end of the Reconstruction era in 1880. Still, it generates the direct decline associated with the loss of the coercion technology.

The third period is characterized by the gradual convergence of Southern mechanization and growth toward the Northern path. Once the coercion channel is shut down, the main asymmetry between the two economies is eliminated; thus, the Southern economy also follows the free-labor sequence of the tuple $\{I_t, \tau_t, v_t\}$ that converges to I^{**}. This adjustment occurs gradually through the differential equations described in (33). As Figure 9b shows, the Southern mechanization rate converges toward the Northern rate over time, although a gap remains throughout the sample. The regional growth rates therefore converge as well, making the South–North GDP-per-worker ratio flatten over time. This pattern is also visible in the historical data toward the end of the sample.

Finally, Figure 11a shows how the quantitative model captures the increase in total innovation over time. Before abolition, horizontal innovation effort is almost zero, as the economy has already reached the profitable frontier of mechanizable tasks. Right after abolition, horizontal R&D spending increases and remains positive. Each new horizontal innovation pushes the set of mechanized tasks beyond I^*, which in turn increases total vertical innovation spending through its effect on the extensive margin. As I_t expands, the set of tasks subject to vertical innovation expands, and total vertical R&D expenditure increases. Postwar increases in mechanization mostly reduce the share of tasks performed by high-skill labor, as shown in Figure 11b. This is due to the reallocation of high-skill labor toward capital production, which increases its wage through the general-equilibrium effect and reduces its direct task share more than that of low-skill labor.[22]

[22]Before abolition, increases in mechanization affected the task allocation through both a direct effect and general-equilibrium effects. The direct effect came from machines replacing coerced low-skill labor at the mechanization margin, causing low-skill labor to lose task share. This reduction in demand for low-skill labor lowered its relative labor cost, making it more competitive in higher-index tasks and pushing $\bar{I}_t$ upward. At the same time, the greater use of high-skill labor in capital production generated an additional general-equilibrium effect by putting upward pressure on high-skill wages, which pushed $\bar{I}_t$ further upward. As a result, both low- and high-skill labor lost direct task shares to mechanization, as shown



(a) Southern R&D Expenditure Shares

(b) Southern Task Shares

Figure 11: Innovation Expenditure and Task Allocation in the South

Overall, the model can quantitatively account for the North–South economic divergence in the United States through the effect of slavery on endogenous innovation incentives. At the same time, it captures the secular increase in the national capital share associated with the rapid mechanization of the economy during the nineteenth century. The model microfounds these mechanisms within a general-equilibrium framework in which the existence of labor coercion alters the incentives to mechanize and, through this channel, affects long-run innovation and growth. The model is also rich enough to show how mechanization affects the allocation of tasks across different types of labor.

4.5 Counterfactual Analysis

Since the model performs well in matching the main targeted moments and captures the evolution of the South–North GDP per worker ratio reasonably well, I next use it to conduct counterfactual exercises examining what would have happened to the Southern economy if abolition had occurred substantially earlier or later in the nineteenth century. The key point is that the economic consequences of abolition depend on its timing. A later abolition implies that the North has already advanced further in mechanization during the age of industrialization, leaving the South with a larger mechanization gap when coercion is eventually removed. Because convergence in mechanization takes place only gradually, a later abolition implies a longer catching-up process and a larger cumulative loss in Southern GDP per worker. Even after the South eventually catches up in terms of mechanization and grows at the same long-run rate as the North, the output lost during the transition is not recovered. Thus, delaying abolition generates a permanent loss in the level of Southern GDP per worker relative to the North. The South also grows more slowly in absolute terms because it spends more time on the lower-mechanization, lower-growth coercive transition path.

To capture and measure these mechanisms quantitatively, I consider counterfactual abolition dates that occur 60, 45, 30, and 15 years earlier than the actual abolition date, as well as one counterfactual in which abolition occurs 15 years later. Table 5 reports the outcomes under these counterfactual scenarios

during the 1800–1820 period in Figure 11b.

in 1910 and compares them with the 1910 outcomes under the benchmark abolition date of 1865. The first row shows that, if abolition had taken place in 1805, 60 years earlier, the South–North real GDP per worker ratio would have been approximately one half. This corresponds to Southern real GDP per worker being 42.8 percent higher in 1910 than under the benchmark abolition date.[23] Thus, the economic gains from early abolition are very large for the Southern economy. These gains become smaller with each 15-year delay in abolition, but remain sizeable for earlier abolition dates. Conversely, if abolition had occurred 15 years later than 1865, Southern real GDP per worker in 1910 would have been about 8 percent lower than under the benchmark 1865 abolition date.

Table 5: Abolition timing and Southern outcomes in 1910, including no abolition

Abolition year	S/N GDP per worker in 1910	Southern GDP per worker vs. benchmark in 1910 (%)	Southern mechanization in 1910 (North = 100)
1805	0.494	42.8	99.99
1820	0.462	33.7	99.96
1835	0.419	21.3	99.71
1850	0.380	9.9	97.88
1865 (benchmark)	0.346	0.0	87.88
1880	0.318	-8.1	73.83
No abolition	0.364	5.2	45.65

Notes: The second outcome column reports the percentage change in Southern GDP per worker in 1910 relative to 1865. Because the Northern GDP-per-worker path is identical across counterfactuals, this is numerically identical to the percentage change in the South–North GDP-per-worker ratio.

An important feature of these counterfactuals is that earlier abolition makes it much more likely that the South catches up with the North in mechanization by 1910. Under the earliest abolition dates, Southern mechanization is almost equal to the Northern level by 1910. In contrast, under the benchmark 1865 abolition date, Southern mechanization remains about 12 percent below the Northern level, while under abolition in 1880 the gap is about 26 percent. These results are due to two factors. First, earlier abolition gives the South more time to catch up with the North before 1910. Second, earlier abolition allows the South to begin transitioning toward the free-labor mechanization frontier earlier while the North is itself still progressing toward that frontier. As a result, the South spends more of the nineteenth century on the high mechanization and higher growth path, which limits the technological gap that emerges between the two regions.

Figure 12 illustrates that the effect of the timing of abolition on Southern relative income is nonlinear. The solid line reports the model-implied South–North GDP-per-worker ratio in 1910 for abolition dates spaced at equal 15-year intervals, while the dashed line connects the 1805 and 1880 outcomes linearly. If the effect of delaying abolition were constant in levels, the model outcomes would lie on this linear benchmark. Instead, the slope is relatively small between 1805 and 1820 and becomes substantially larger for delays during the middle of the nineteenth century.

To account for and quantify these nonlinearities in relative income, I perform the following decomposition exercise, where T denotes the counterfactual abolition date. The overall change in Southern GDP per worker relative to the North between 1800 and 1910 can be decomposed as

[23] Because the Northern real GDP-per-worker path does not change across the counterfactual dates, differences in the South–North ratio in 1910 are driven solely by the change in Southern real GDP per worker itself.

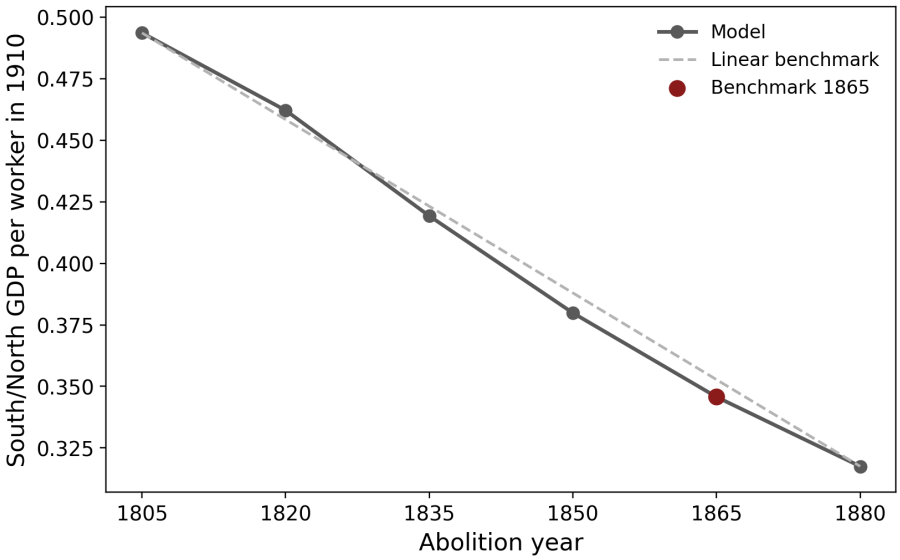


Figure 12: Counterfactual Abolition Dates

$$\Delta_{1800,1910} \log\left(\frac{Y^S(T)}{Y^N}\right) = \underbrace{\Delta_{1800,T} \log\left(\frac{Y^S(T)}{Y^N}\right)}_{\text{pre-abolition relative growth}} + \underbrace{\Delta_T \log\left(\frac{Y^S(T)}{Y^N}\right)}_{\text{immediate abolition effect}} + \underbrace{\Delta_{T,1910} \log\left(\frac{Y^S(T)}{Y^N}\right)}_{\text{post-abolition relative growth}}.$$

The first term measures the change in Southern GDP per worker relative to the North between 1800 and the abolition date, and therefore captures the relative growth loss accumulated while the South remains under coercion. The second term captures the discrete change due to the immediate coercion productivity loss at abolition. The third term measures the change in Southern GDP per worker relative to the North in post-abolition period until 1910.[24]

These components differ across abolition dates because changing T changes both the length of time the Southern economy remains on the low-growth coercive path and the mechanization rate of the Southern economy when abolition occurs. The decomposition in Table 6 reports the contribution of each component at different abolition dates. The decomposition shows that the source of the loss changes substantially with the timing of abolition. The first component, reported in the first column, becomes larger for later abolition dates because the South spends more time on the slower coercive

[24]More rigorously, these terms correspond to the following mathematical decomposition:

$$\underbrace{\left[\log\left(\frac{Y^S_{1800}(T)}{Y^N_{1800}}\right) - \log\left(\frac{Y^S_{1910}(T)}{Y^N_{1910}}\right)\right]}_{\text{total relative GDP-per-worker loss}} = \underbrace{\left[\log\left(\frac{Y^S_{1800}(T)}{Y^N_{1800}}\right) - \log\left(\frac{Y^{S,\text{pre}}_T(T)}{Y^N_T}\right)\right]}_{\text{pre-abolition relative-growth loss}}$$

$$+ \underbrace{\left[\log\left(\frac{Y^{S,\text{pre}}_T(T)}{Y^N_T}\right) - \log\left(\frac{Y^{S,\text{post}}_T(T)}{Y^N_T}\right)\right]}_{\text{immediate abolition loss}} + \underbrace{\left[\log\left(\frac{Y^{S,\text{post}}_T(T)}{Y^N_T}\right) - \log\left(\frac{Y^S_{1910}(T)}{Y^N_{1910}}\right)\right]}_{\text{post-abolition relative-growth loss}}.$$

transition path while the North continues along the free-labor mechanization path. With abolition in 1805, the South accumulates only 2.48 log points of pre-abolition relative-growth loss, since it spends very little time on the coercive transition path. As abolition is delayed, the accumulated pre-abolition loss rises sharply: it reaches 28.09 log points in 1850 and 37.97 log points in the 1865 benchmark. The second component is the immediate productivity loss and is larger for very early abolition dates because the Southern economy is then less mechanized and more labor intensive, making the removal of the productivity advantage associated with coercion more costly on impact. It starts at 30.61 log points for abolition in 1805, declines slightly, and then stabilizes at about 27.94 log points.

Table 6: Decomposition of Southern relative GDP-per-worker losses by abolition date

Abolition year	Pre-abolition relative-growth loss (log points)	Immediate abolition loss (log points)	Post-abolition relative-growth loss to 1910 (log points)	Post-abolition relative-growth loss over 30 years (log points)
1805	2.48	30.61	4.47	2.09
1820	10.25	28.17	5.73	3.49
1835	18.72	27.96	7.21	4.89
1850	28.09	27.94	7.73	5.54
1865 (benchmark)	37.97	27.94	7.26	5.71
1880	47.99	27.94	5.74	5.74
No abolition	68.09	–	–	–

Notes: Entries are losses in Southern GDP per worker relative to the North, expressed in log points. Each entry is constructed as minus the corresponding change in the log South–North GDP-per-worker ratio, multiplied by 100. Hence, positive values indicate a decline in Southern relative GDP per worker. For each abolition date, the pre-abolition relative-growth loss, the immediate abolition loss, and the post-abolition relative-growth loss through 1910 sum to the total relative GDP-per-worker loss between 1800 and 1910, up to rounding. The final column reports the post-abolition relative-growth loss over a common 30-year horizon and is not an additional component of the 1910 decomposition. In the no-abolition row, coercion remains in place through 1910, so the first column reports the entire 1800–1910 relative-growth loss and the abolition and post-abolition entries are not applicable.

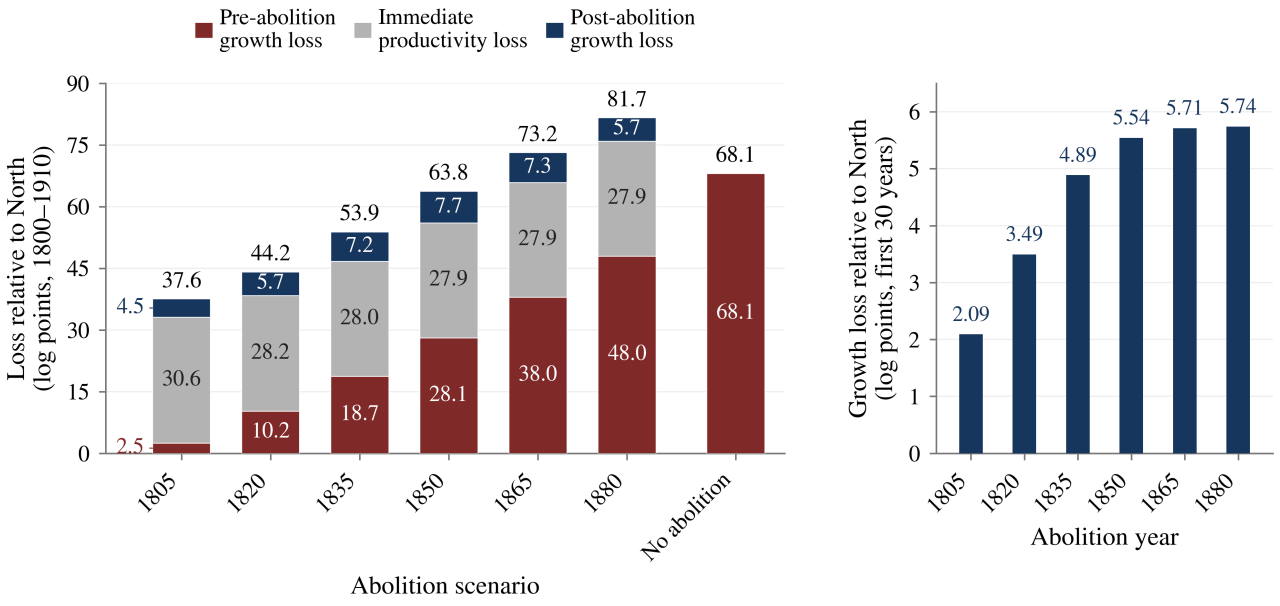
The third component is the post-abolition relative-growth component, reported in the third column. It shows that the South does not immediately attain the same growth rate as the North after abolition, since a mechanization gap still remains. This component therefore also depends on the state of mechanization at the time of abolition. Earlier abolition leaves the South with a smaller mechanization gap relative to the North and therefore limits the subsequent relative-growth loss. For example, following abolition in 1805, the post-abolition relative GDP-per-worker loss through 1910 is only 4.47 log points, whereas it is substantially larger for later abolition dates. This effect appears to peak around 1850 and then becomes less severe. This decline, however, partly reflects the fixed 1910 endpoint. For example, for the abolition in 1880, the post-abolition period observed before 1910 is only 30 years, leaving less time for the post-abolition relative-growth effect to accumulate.

To account for differences in the length of the post-abolition period, the last column reports the change in relative GDP per worker over the first 30 years after abolition for each counterfactual date. [25]

[25] Specifically, I compute the following term to compare the post-abolition growth differential over an equal-length horizon across all abolition dates:

$$\underbrace{\Delta_{T,T+30} \log\left(\frac{Y^S(T)}{Y^N}\right)}_{\text{post-abolition relative growth over the first 30 years}} .$$

Figure 13 illustrates the decomposition in Table 6. Panel (a) shows the decomposition of growth losses into three different components; panel (b) compares subsequent growth losses over the first 30 years after emancipation.



(a) Growth and productivity losses

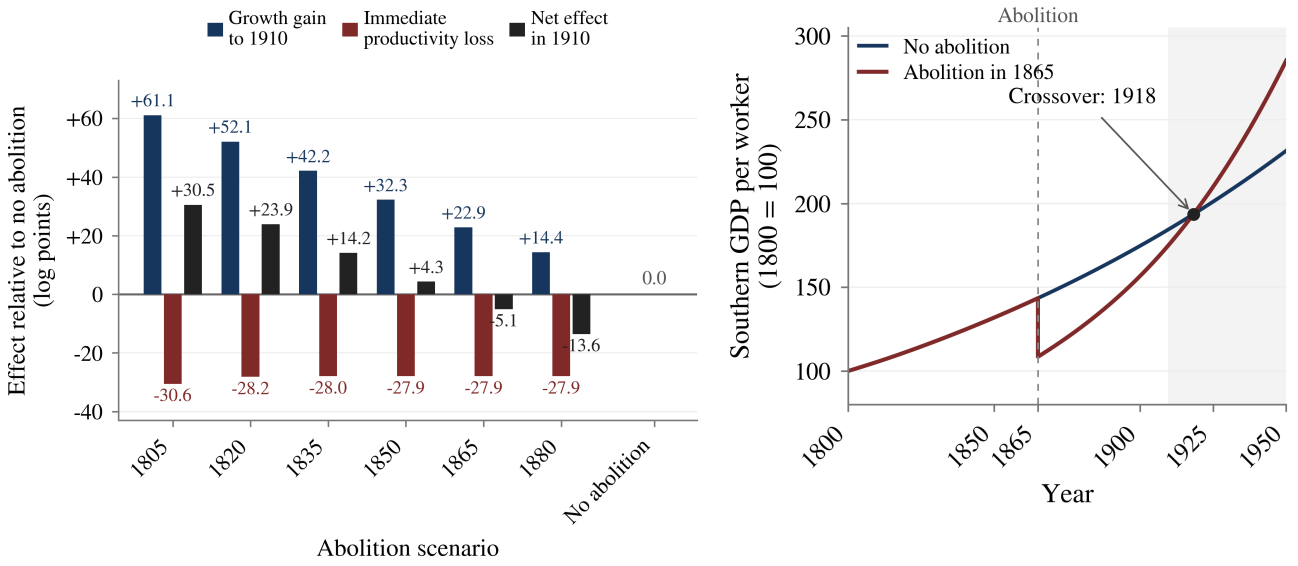
(b) First 30 years after abolition

Figure 13: Decomposition of Southern GDP-per-worker losses by abolition date

As predicted by the theory, once the length of the post-abolition period is held constant, later abolition produces a larger growth gap between the North and the South. As the last column of Table 6 and panel (b) of Figure 13 show, the post-abolition relative-growth loss over this common 30-year horizon increases monotonically as abolition is delayed.

Figure 14 compares these losses with the income path under continued slavery. Panel (a) shows the cumulative growth gains from abolition by 1910, the immediate productivity loss, and their net effect, expressed in log points relative to no abolition by 1910. Panel (b) shows why the endpoint matters: although continued slavery yields Southern GDP per worker 5.2 percent higher than the 1865 benchmark in 1910, the abolition path overtakes it around 1918 and is 23.3 percent higher by 1950. The paths beyond 1910 extend the same calibrated model and show how growth gains from abolition eventually outweigh the initial productivity loss.

Overall, using the estimated parameters, the model generates substantial economic gains for the Southern United States from earlier rather than later abolition. These gains arise mainly because earlier abolition reduces the time spent on the low-growth path under slavery. In contrast, later abolition leads to a larger gap in levels and slower convergence of growth rates in the post-abolition period.



(a) Gains relative to no abolition

(b) 1865 abolition and no abolition

Figure 14: Abolition and Southern GDP per worker

5 Discussion

In the model, coercion either exists or does not, with $c(i) > 1$ or $c(i) = 1$, respectively. But the post-abolition period, especially after Reconstruction, was still characterized by Jim Crow laws that maintained the oppression of Black Americans. To consider this, as in the two-household model in the appendix, coercion can be represented as a continuum rather than as something that either exists or does not, using the term $c(i)^{\theta}$, where $\theta = 1$ represents full coercion under slavery and $\theta = 0$ represents no coercion at all. Thus, the post-abolition period can be considered a decline from $\theta = 1$, or full coercion, to partial coercion, with θ somewhere between 0 and 1. This would not change the qualitative results of the model. There is still a decline in coercion that opens new tasks to mechanization, increasing innovation and the post-abolition growth rate of the Southern economy while also creating an immediate productivity loss. But quantitatively, the immediate productivity decline could be smaller in this case, as some coercion still exists. The range of tasks the South mechanizes could also be narrower than in the case without coercion. Thus, quantitatively, this case would yield a smaller immediate productivity decline but also lower long-run growth than the complete removal of coercion.

Second, the framework features individual tasks as the smallest units of production without explicitly considering broad sectors such as agriculture, manufacturing, and services or structural change. As in Jones and Liu (2024), tasks can be grouped into broad sectors such as agriculture, manufacturing, and services, where automation is the primary source of structural change, shifting labor toward the remaining tasks as others become automated. In this paper, if agricultural tasks are grouped among

the low-index tasks, then automating those tasks would shift labor toward higher-index tasks, such as those in manufacturing. Thus, a failure to mechanize agricultural tasks under slavery could explain the limited shift in employment toward manufacturing tasks and the slower overall structural change in the South.

6 Conclusion

This paper argues that slavery, by weakening incentives to innovate, can explain the South's economic decline relative to the North in the nineteenth century. The paper provides empirical evidence on this mechanism, documenting limited patenting before emancipation in technologies targeting tasks performed by enslaved workers. Following emancipation, patenting in these technologies increased approximately threefold relative to patenting in technologies targeting similar tasks performed by free workers.

To explain these patterns and their implications for aggregate growth, the paper develops a task-based endogenous growth model in which coercion acts as a technology that raises labor productivity. Although this generates static productivity gains, it also weakens incentives to develop labor-saving machines. Since mechanization opens tasks to subsequent improvements in machine quality, weaker incentives to mechanize also reduce long-run growth. The quantitative model shows that these dynamic losses generated substantial long-run costs for the Southern U.S. economy. In the most striking case, abolition 60 years earlier would have increased Southern GDP per worker by 43 percent in 1910. The estimated model shows that emancipation in 1865 initially caused a substantial decline in Southern GDP per worker. However, subsequent innovation gains eventually outweighed this initial loss, with Southern GDP per worker exceeding the level under continued slavery roughly 50 years after abolition. These results highlight the paper's central trade-off between static productivity gains and dynamic growth losses: while coercion can raise current output by increasing labor effort, its effects on innovation can eventually outweigh this initial productivity advantage.

One limitation of the quantitative analysis is that the model does not explicitly account for the direct effects of the Civil War, such as capital destruction and disruptions to financial markets. These factors may have affected the observed outcomes of the Southern economy in the years following the Civil War. The counterfactual exercises capture the effects of abolition through the mechanisms within the model, while abstracting from the direct economic consequences of the war. Future work could incorporate these wartime shocks to better account for the short-run evolution of Southern income around the Civil War and emancipation.

More broadly, the paper shows how labor market institutions can affect long-run growth by changing incentives to develop and improve technologies. While I focus on slavery, a similar framework could be used to study collective bargaining or sectors with monopsony power and their impact on automation and innovation in a similar setup, connecting these effects to aggregate long-run growth. Moreover, this framework could also be used to study how the large refugee inflows of the 2010s affected firms' technology adoption and machinery investment decisions in host countries, focusing on sectors and tasks where refugees were employed informally. These applications will be the next steps in my research agenda.

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A Data and Empirics

A.1 Agricultural Capital Shares: Measurement and Regressions

Capital-share measurement. The main analysis of capital shares in Figure 3 uses agricultural capital shares in 1850 and 1860, but I also construct a panel through 1900 for the adoption analysis in Appendix A.8. Agricultural values come from Haines et al. (2018), and population counts from Haines and Inter-university Consortium for Political and Social Research (2010). I calculate the land-and-capital share using Gallman's gross capital return procedure:

$$\text{Capital Share} = \frac{\begin{array}{c}0.05 \cdot \text{Land Value} + 0.07 \cdot \text{Buildings} \\ + 0.10 \cdot \text{Equipment} + 0.05 \cdot \text{Inventories}\end{array}}{\text{Value of Output}}. \tag{A-1-1}$$

The coefficients are the return rates used by Gallman (1972, p. 205, n. 29) to compute the aggregate agricultural capital and labor shares in the U.S. He set the rates of return for each asset as follows: a 5% net return on land and inventories, and gross returns of 7% on buildings and 10% on equipment. Gross returns also include depreciation, with a higher allowance for short-lived equipment than for buildings. Gallman uses gross returns because his income measure is gross of capital consumption. He also holds these rates constant over time.

Inventories comprise living livestock plus half the annual output value of fourteen products: grains, potatoes, peas and beans, butter, cheese, wool, and slaughtered animals. The capital-only measure excludes land. Buildings are separated from land using the state-level ratios of Primack (1962) through 1890; the 1900 census reports building values separately.

The reconstructed 1850 output is converted from 1860 to 1850 prices using the factor 71/77 from the farm-products wholesale price index in U.S. Department of Agriculture (1965, p. 13). For the panel through 1900, reported census output is used from 1870 onward; product inventories are valued at 1860 commodity prices and revalued using the same index. Missing slaughter values in 1880 and 1890 are imputed from livestock values using the pooled national slaughter-to-livestock value ratio in 1850–1860.[26]

National comparison. Once I construct my estimated capital shares, I can compare them at the national level with those estimated by Gallman. Rather than using the census data directly, Gallman measures shares relative to agricultural gross income: farm gross product from Towne and Rasmussen (1960) plus rent paid to nonfarm landlords. This differs in coverage and intermediate-input treatment from census output. Table 7 compares the national capital shares excluding land. The estimates are close to Gallman's, especially when the Towne–Rasmussen income denominator is used: the difference is less than 0.65 percentage points in every census year from 1850 to 1900. For 1850 and 1860, the years of primary interest, the estimates are already close using census output and become closer using the common income denominator. Applying the same annual national output correction coefficient to

[26] In this extended panel, the 1850 price adjustment applies only to reconstructed output values; other components, such as farm equipment, are already valued at current prices in the census. The original prewar regressions scale the full reconstructed 1850 output by 71/77. The inventory extensions approximate Gallman's method.

each county leaves the regression results mostly unchanged. Thus, I use the census output directly and apply the gross return procedure at the state and county levels to make the capital-share comparisons below.

Table 7: U.S. Agricultural Capital Shares: Census and Gallman, 1850–1900

	Income		Capital share		
	Millions of current dollars		Percent		
Year	Census	Adjusted	Census	Adjusted	Gallman
1850	993	933	10.32	10.98	11.10
1860	1,649	1,592	11.81	12.23	12.70
1870	2,448	2,727	11.85	10.64	11.10
1880	2,213	3,170	14.30	9.98	10.60
1890	2,460	3,242	16.81	12.76	13.40
1900	3,742	4,058	14.56	13.43	13.20

Notes: Capital shares exclude land. Census income is reconstructed census output in 1850–1860 and reported farm output thereafter. Adjusted income is broad farm gross product from Towne and Rasmussen (1960, Tables 1 and 3) plus rent paid to nonfarm landlords, as used by Gallman (1972, pp. 194–195). Adjusted shares hold census-based capital returns fixed and use adjusted income as the denominator. Gallman's shares are from his Table 5. Return rates follow equation (A-1-1). Inventories use fourteen products, with slaughter imputed in 1880 and 1890. Coverage, valuation, and census-versus-calendar-year timing differ between the series.

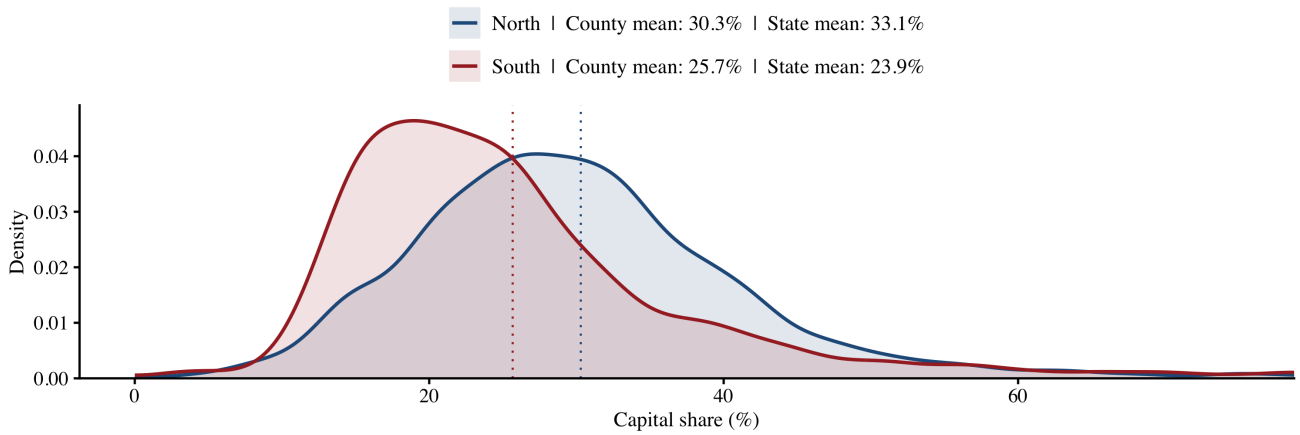


Figure 15: County Distribution of Land and Capital Shares: North and South, 1860

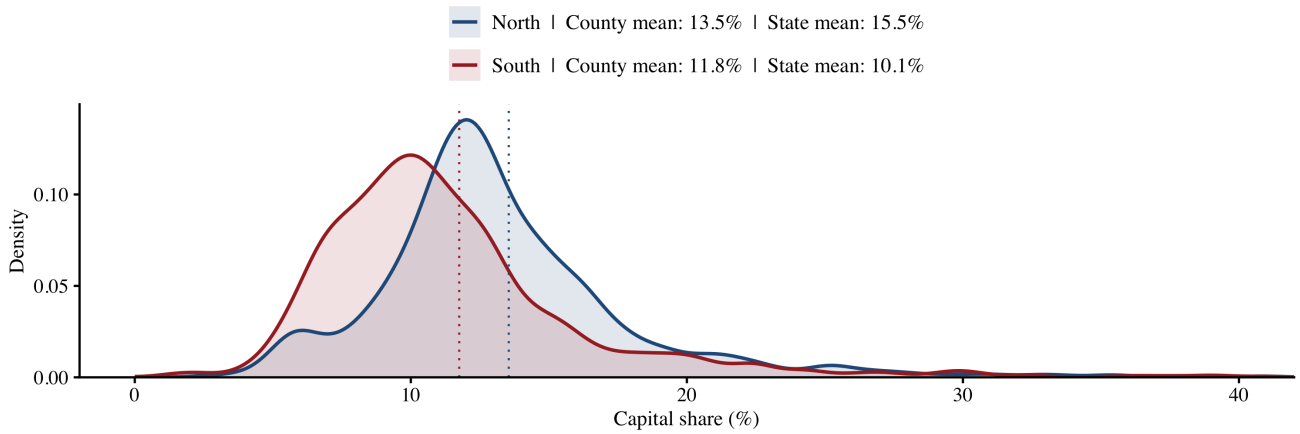


Figure 16: County Distribution of Capital Shares Excluding Land: North and South, 1860

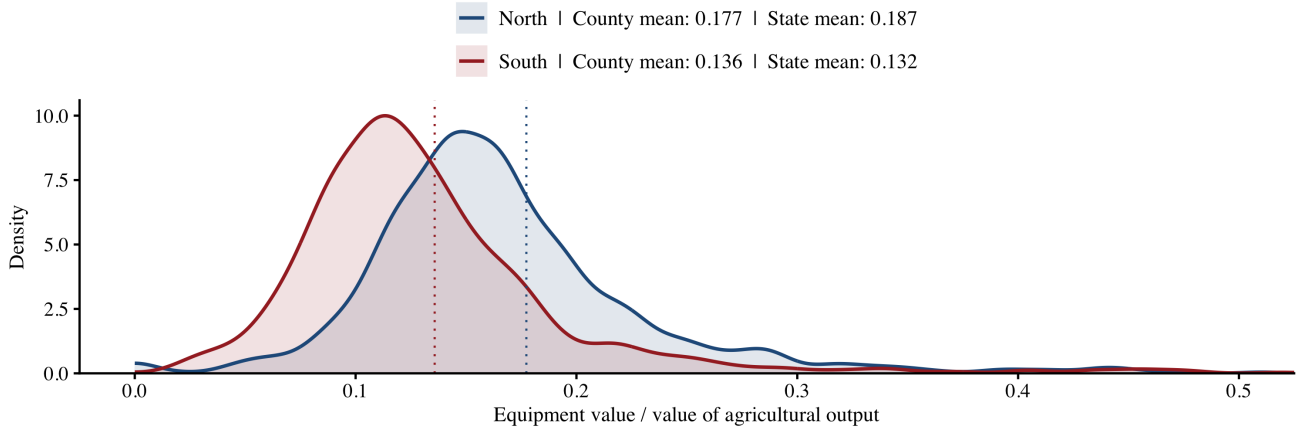


Figure 17: County Distribution of Equipment Value Relative to the Value of Agricultural Output: North and South, 1860

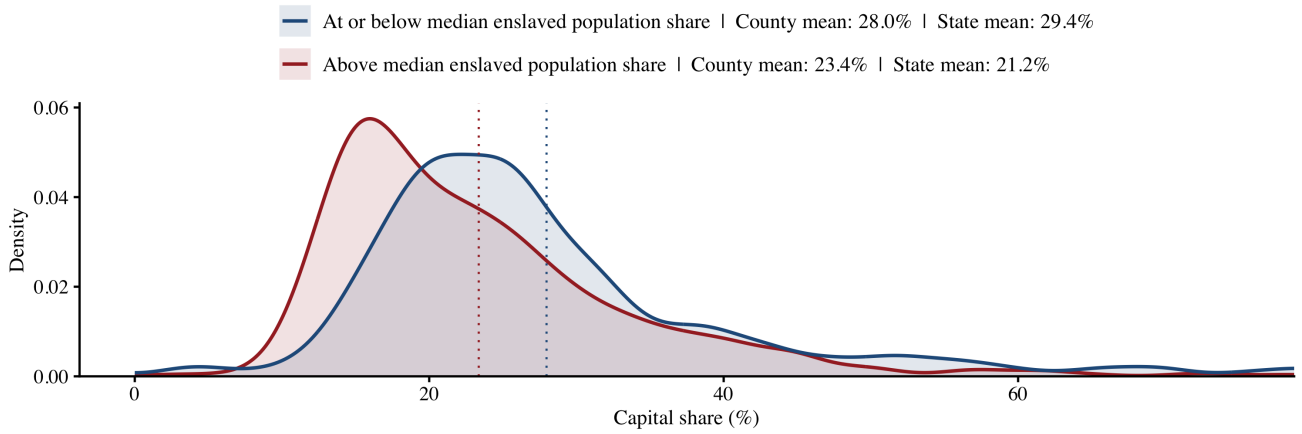


Figure 18: County Distribution of Land and Capital Shares: High versus Low Enslaved Population Shares, South, 1860

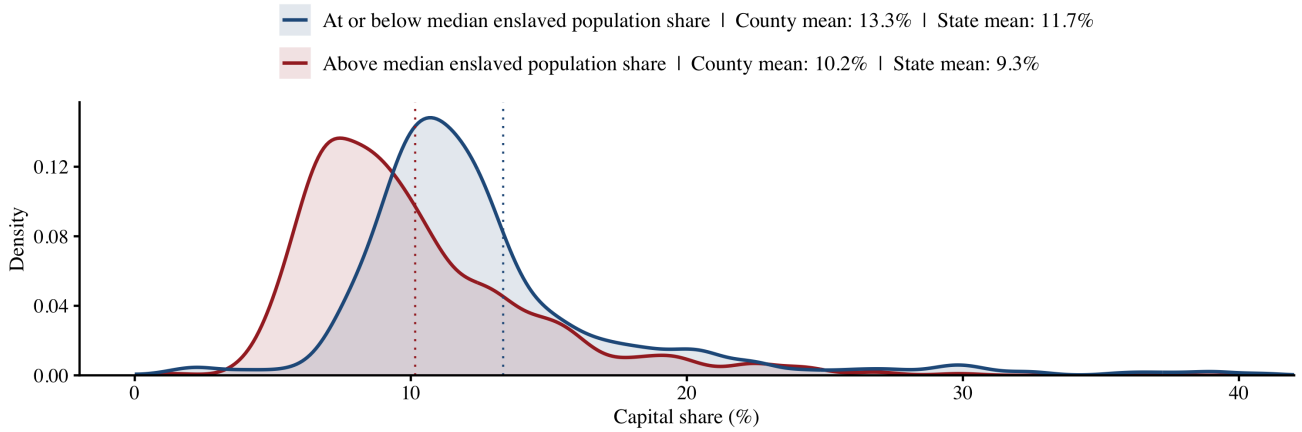


Figure 19: County Distribution of Capital Shares Excluding Land: High versus Low Enslaved Population Shares, South, 1860

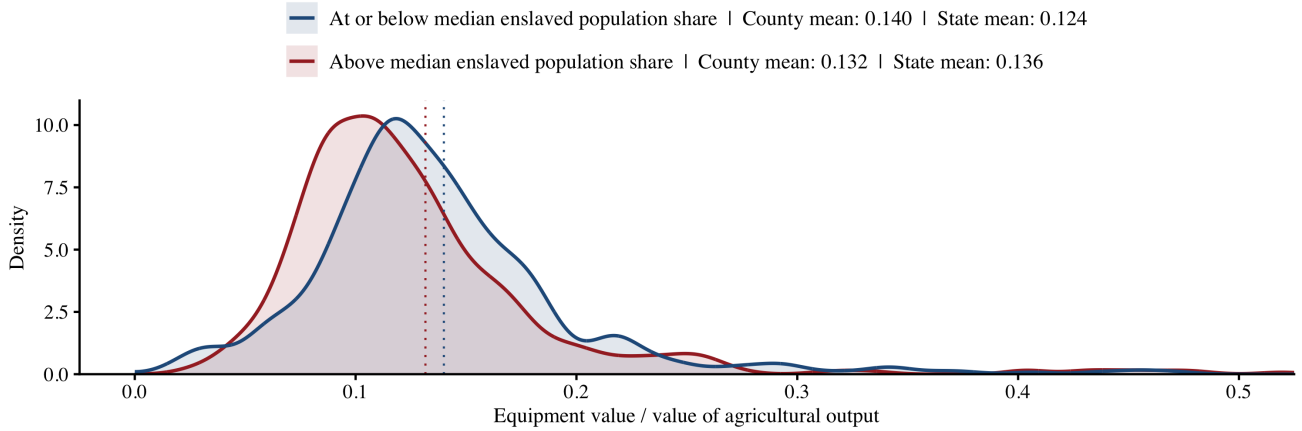


Figure 20: County Distribution of Equipment Value Relative to the Value of Agricultural Output: High versus Low Enslaved Population Shares, South, 1860

Sample and regression specification. The regressions use Southern states and counties in 1850 and 1860 with complete inputs, nonnegative reported asset values, positive output, and valid enslaved-population shares. The District of Columbia is excluded because its land-to-building ratio is unavailable. County observations with either capital-share measure above 100% are excluded. Both outcomes use the same sample at each geographic level: 30 state-year observations and 2,034 county-year observations across 15 states.

For each outcome and geographic level, I estimate the unweighted OLS regression

$$\text{Capital Share}_{it} = \alpha + \beta\,\text{Slave Share}_{it} + \delta_t + \varepsilon_{it},$$

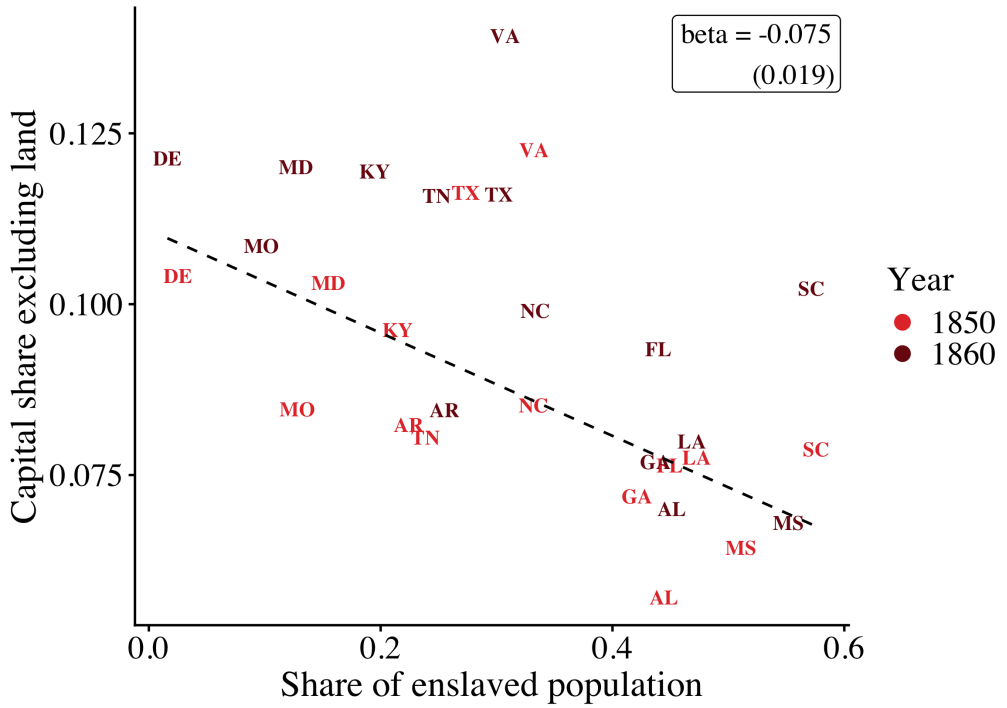
where i indexes states or counties, $t \in \{1850, 1860\}$, $\text{Capital Share}_{it}$ is one of the two measures defined above, and Slave Share_{it} is the enslaved population divided by total population, including free and enslaved residents. Both shares are fractions. The term δ_t denotes year fixed effects and ε_{it} is the regression residual. The specification includes no state or county fixed effects. These regressions describe associations rather than causal effects. The dashed lines in the state-level scatterplots show the fitted relationship from this specification, evaluated at the 1850 year effect.

Regression results. Table 8 shows a negative association between enslaved population shares and agricultural capital shares at both the state and county levels, whether land is included or excluded.

Table 8: Enslaved Population and Agricultural Factor Shares, 1850–1860

	States		Counties	
	Land and capital	Capital only	Land and capital	Capital only
	(1)	(2)	(3)	(4)
Enslaved population share	-0.247***	-0.075***	-0.092***	-0.067***
	(0.055)	(0.019)	(0.011)	(0.006)
Year fixed effects	Yes	Yes	Yes	Yes
Observations	30	30	2034	2034
States	15	15	15	15
R^2	0.506	0.447	0.089	0.091
Mean dependent variable	0.215	0.094	0.231	0.109

Notes: Conventional OLS standard errors are in parentheses. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$, using two-sided t-tests.



Southern states, 1850 and 1860. Dashed line: fitted values holding year at 1850.
1850 output adjusted by 71/77. Parentheses report conventional OLS standard errors.

Figure 21: Agricultural Capital Shares Excluding Land and Enslaved-Population Shares in Southern States, 1850–1860

A.2 Geographic Distribution of Agricultural Production

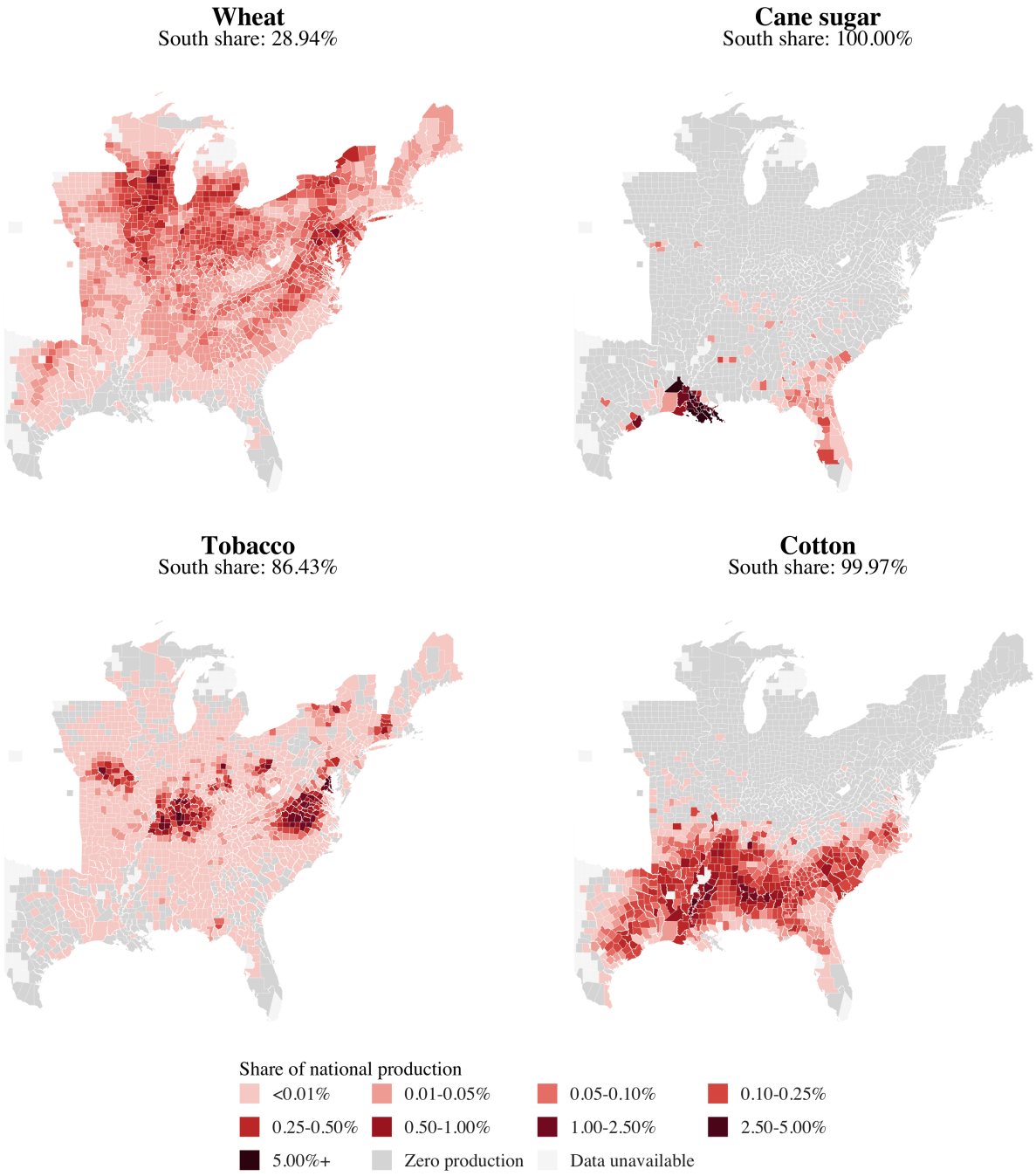
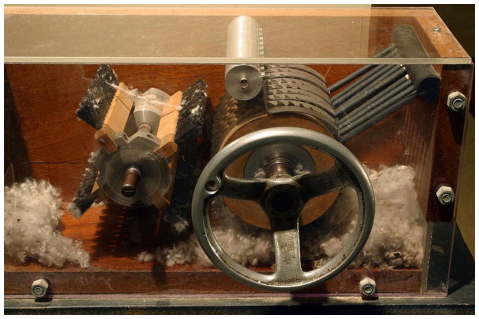


Figure 22: County Shares of National Crop Production, 1860 Census

Notes: Production refers to the 1859 harvest. Southern shares use reported state totals and include Missouri and the border slave states.

A.3 Innovations in Agricultural Implements



Eli Whitney cotton gin (1793)



McCormick mechanical reaper (1831)



John Deere steel plow (1837)

X8277

UNITED STATES PATENT OFFICE.

CYRUS H. McCORMICK, OF ROCKBRIDGE COUNTY, VIRGINIA.

IMPROVEMENT IN MACHINES FOR REAPING SMALL GRAIN.

Specification forming part of Letters Patent dated June 21, 1834

To all whom it may concern:

Be it known that I, CYRUS H. McCORMICK, of Rockbridge county and State of Virginia, have invented a new and useful Improvement in Reaping all Kinds of Small Grain; and I do hereby declare that the following is a full and exact description of the construction and operation of the said machine as invented or improved by me.

Upon a frame of wood is to be constructed a platform of about six feet in width by about four or five in length. From the back of this of inserting shafts, so that the machine may be drawn by one horse in this manner. The two head-pieces are to be lengthened, as also the curved brace projecting toward all of them, about three or four feet. The two broad pieces will be connected at their ends by a bar for the single-tree, and, rising from the right-hand one near the end, an upright connects it with the curved brace, and by the side of this upright rises another, secured to its place to a height sufficient to clear the reel. From this top a brace passes across the reel to the oppo-

Cyrus McCormick Patent

United States Patent Office.

JOHN DEERE, OF MOLINE, ILLINOIS, ASSIGNOR TO DEERE & CO., OF SAME PLACE.

Letters Patent No. 63,369, *dated April* 2, 1867.

IMPROVED METHOD OF MAKING PLOUGHS.

The Schedule referred to in these Letters Patent and making part of the same.

TO ALL WHOM IT MAY CONCERN:

Be it known that I, JOHN DEERE, of Moline, in the county of Rock Island, and State of Illinois, have invented certain new and useful improvements in the Manufacture of Ploughs; and I do hereby declare that the following is a full, clear, and exact description thereof, reference being had to the accompanying drawings, making part of this specification, and to the letters of reference marked thereon, like letters indicating like parts wherever they occur.

To enable others skilled in the art to construct and use my invention I will proceed to describe it.

My invention has relation to the manufacture of ploughs made of steel or wrought iron, and consists in the construction and use of certain tools hereinafter described for the purpose of making the ploughs uniform in shape and style.

Figure 1 is a top plan view of the land-side and share, with the tools applied for welding the land-side and share together.

Figure 2 is a rear view of the same.

Figure 3, a view of the welding-brace detached; and

Figures 4 and 5 are views of the templet shown detached.

It is well known that the land-side and shares of this style of ploughs are the parts most subjected to wear; and that it is difficult in replacing them to give to them precisely the same angles and position which the originals

John Deere Steel Plow Patent

Figure 23: Innovations in Agricultural Implements

Corn-harvesting technologies are classified as mixed because corn was produced extensively in both regions, where harvesting machinery could replace both enslaved and free labor. Using reported state totals from the 1860 agricultural census, the South accounted for 52.1% of national corn production and the North for 47.9% (Haines et al., 2018). These figures refer to the 1859 harvest and use the paper's definition of the South, including the border slave states and Missouri.

Table 9: USPC-Based Classification of Patent Subclasses by Relationship to Slave and Free Labor

Patents are assigned to four groups using **USPC** codes based on their relationship to slave vs. free labor:

Group	**Label**	**USPC Codes**
G1	Slave Replacing	Cl. 56 (sub. 27.5, 28–50, 62); Cl. 131 ($\geq$280 $\leq$ 327). Cotton picking& harvesting, tobacco & sugarcane harvest, tobacco processing.
G2	Mixed Replacing	Cl. 56 (51–121, excl. 62); Corn Harvesting.
G3	Slave Complementary	Cl. 19 (1–67). Cotton ginning.
G4	Free-Labor Replacing	Cl. 56 ($<$27.5, $>$121); Cl. 172, 460, 111. Grain/hay harvest, plows, threshing, drills.

A.4 Geographic Validation of the Patent Classification

This section uses patent locations to validate the geographic consistency of the slave- and free-labor-replacing technology groups.

Geographic measures. Using pre-1865 patents with observed locations, I compare three measures of Southern orientation. The *patent-weighted Southern share* is the fraction of patents in each technology group that originated in the South. The *mean subclass Southern share* averages this fraction across subclasses with observed patents, giving each subclass equal weight. The third measure repeats this average using only subclasses with at least two pre-1865 patents, reducing the influence of singleton subclasses.

Table 10: Geographic Validation of Technology Groups before Emancipation

Measure	Slave-replacing	Mixed	Free-labor
Patent-weighted Southern share	46.7%	22.1%	13.5%
Mean subclass Southern share	51.0%	16.0%	13.8%
Mean subclass share, ≥ 2 patents	38.1%	29.6%	14.5%

Table 10 shows the expected ordering of Southern patent shares under all three measures:

$$\text{Free-labor} < \text{Mixed} < \text{Slave-replacing}.$$

Slave-replacing patents have the greatest Southern orientation, free-labor patents are predominantly Northern, and mixed technologies lie between the two.

Geographic regression. I estimate a linear probability model for whether a pre-1865 patent originated in the South:

$$\text{South}_{ijt} = \alpha_t + \beta_1 \,\text{Slave-replacing}_j + \beta_2 \,\text{Mixed}_j + \varepsilon_{ijt}.$$

Here i indexes patents, j subclasses, and t patent years; South_{ijt} equals one for Southern patents and zero for Northern patents. Year fixed effects absorb common changes in patent origins. Free-labor technologies are the reference group in column (1); column (2) uses mixed technologies as the reference group.

Table 11: Technology Type and Southern Patent Origin before Emancipation

Dependent variable	Southern patent origin	
	(1)	(2)
Slave-replacing	0.3204^{***}	0.2384^{*}
	(0.1204)	(0.1270)
Mixed	0.0820^{*}	
	(0.0427)	
Free-labor		-0.0820^{*}
		(0.0427)
Year fixed effects	Yes	Yes
Observations	3,661	3,661
Omitted reference group	Free-labor	Mixed

Notes: The sample includes pre-1865 patents with observed locations in the three technology groups. Standard errors clustered by USPC subclass are in parentheses. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Together, these results support the grouping: slave-replacing patents are more likely to originate in the South, while free-labor-replacing patents are predominantly Northern.

A.5 Descriptive Statistics for Patent Innovation

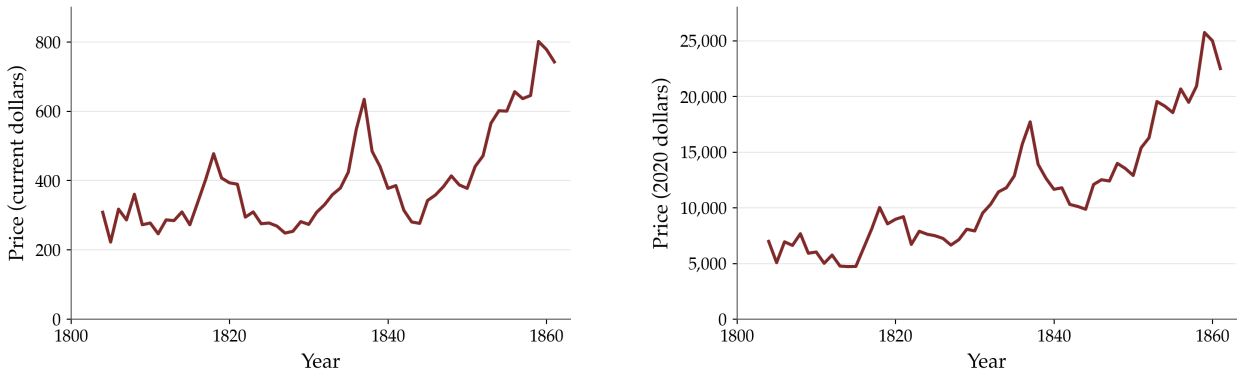
Table 12: Descriptive Statistics: Patent Innovation

Technology group	Period	Subclasses	Subclass-bin count	Mean	Std. dev.	Zero (%)
Panel A. Patents per subclass and five-year bin						
Slave-labor replacing	All periods	61	854	1.13	2.27	59.6
Free-labor replacing	All periods	1,382	19,348	2.10	4.27	47.8
Slave-labor replacing	Pre-1865	61	244	0.12	0.43	91.4
Free-labor replacing	Pre-1865	1,382	5,528	0.62	2.02	76.8
Slave-labor replacing	Post-1865	61	610	1.54	2.56	46.9
Free-labor replacing	Post-1865	1,382	13,820	2.68	4.77	36.2
Panel B. $\log(1 + patents)$						
Slave-labor replacing	All periods	61	854	0.48	0.66	59.6
Free-labor replacing	All periods	1,382	19,348	0.71	0.82	47.8
Slave-labor replacing	Pre-1865	61	244	0.07	0.25	91.4
Free-labor replacing	Pre-1865	1,382	5,528	0.26	0.54	76.8
Slave-labor replacing	Post-1865	61	610	0.64	0.71	46.9
Free-labor replacing	Post-1865	1,382	13,820	0.89	0.85	36.2

Notes: Each observation is a patent subclass by five-year bin; zero-count cells are included. Pre-1865: 1845-1864. Post-1865: 1865-1914. The logged measure is the natural log of one plus patents. Poisson estimates use raw patent counts; Panel B is descriptive.

A.6 Historical Slave Prices

Figure 24: Average Slave Purchase Prices, 1804–1861

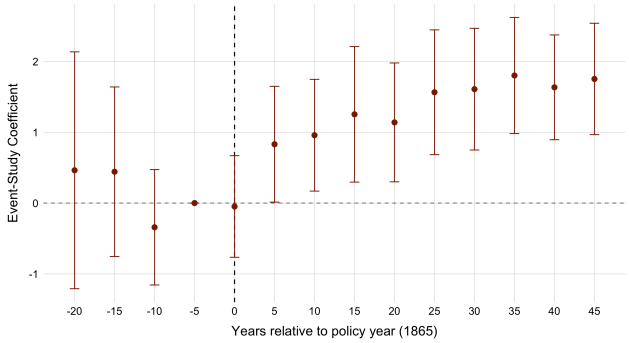


(a) Current dollars

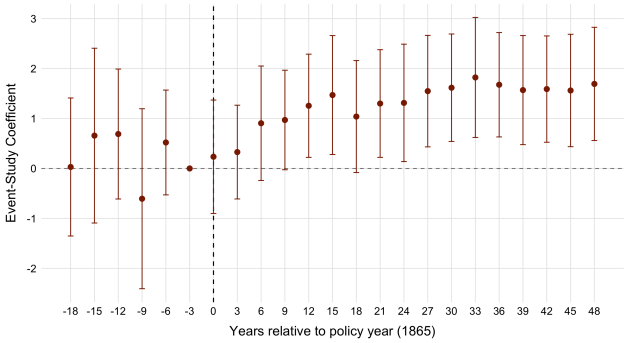
(b) Constant 2020 dollars

Notes: Average purchase prices per enslaved person from *Historical Statistics of the United States*, Millennial Edition (2006), series Bb212. Real prices use the annual U.S. CPI from MeasuringWorth (Officer and Williamson), with each current-dollar price multiplied by the ratio of the 2020 CPI to that year's CPI.

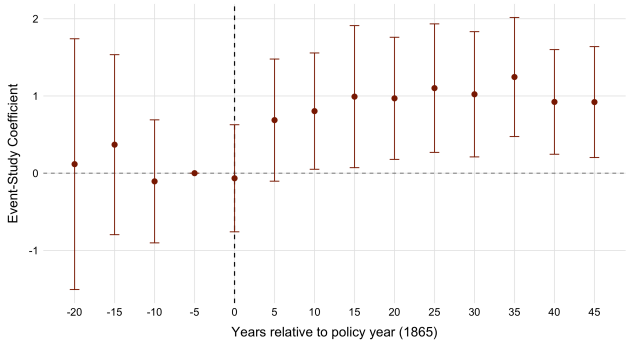
Figure 25: Robustness of the Patent Event-Study Estimates



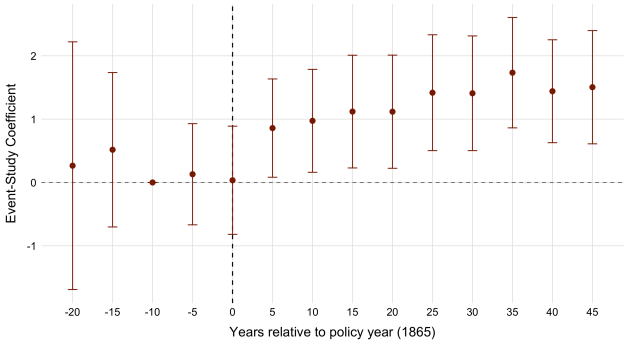
(a) Hay and grain harvesting controls



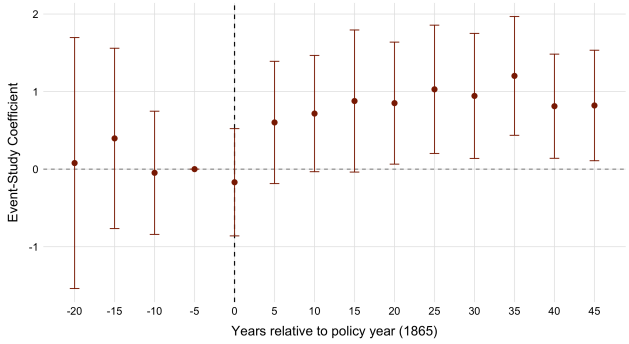
(b) Three-year bins



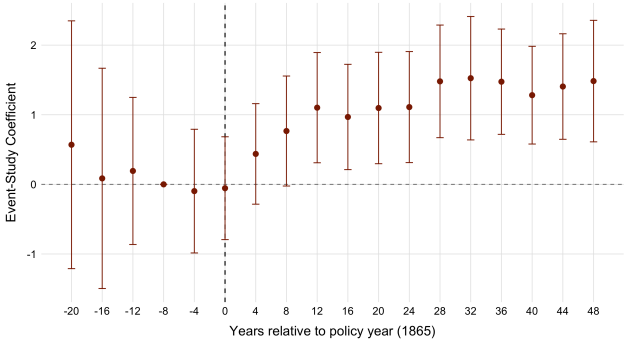
(c) Agriculture, husbandry, and food controls



(d) Five-year bins; reference: 1855–1859



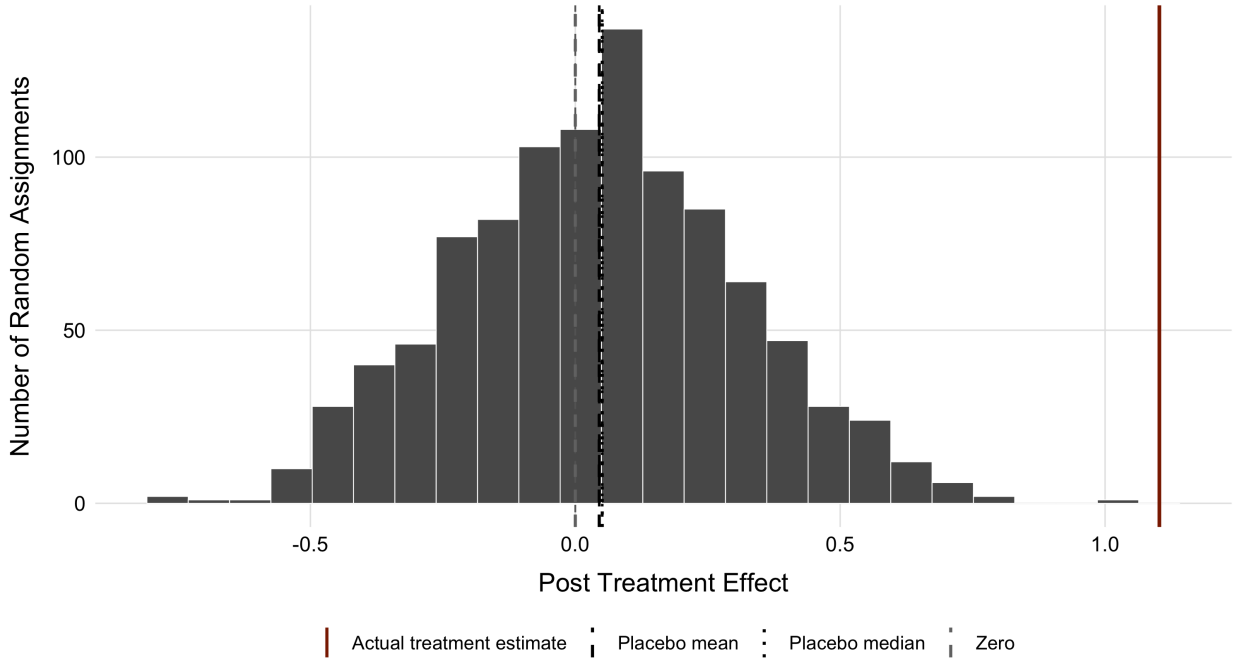
(e) Expanded controls: NBER 61 and 64



(f) Four-year bins; reference: 1857–1860

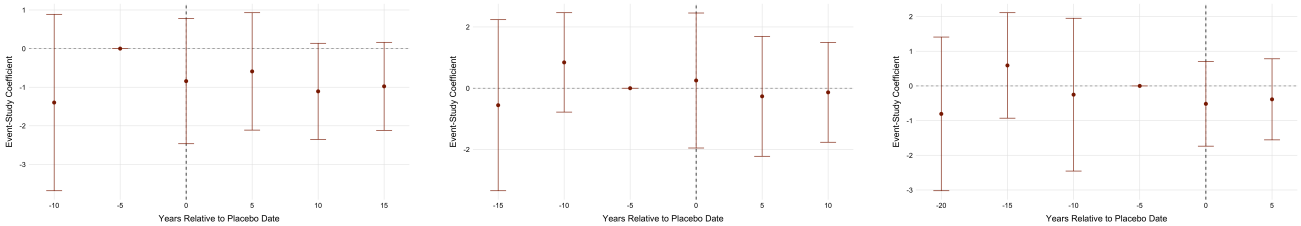
Notes: Left panels vary the control group; expanded controls add earth working and wells. Right panels vary the bin length or reference period. All panels report Poisson event-study coefficients with subclass and period fixed effects.

Figure 26: Placebo Test: Random Treatment Assignment within the Main Sample



Notes: The histogram reports estimates from 1,000 random assignments of 61 treated subclasses within the main sample. The red line marks the actual Poisson difference-in-differences estimate.

Figure 27: Placebo Event Studies with Pre-Emancipation Treatment Dates



(a) Placebo date: 1845 (b) Placebo date: 1850 (c) Placebo date: 1855

Notes: Each panel assigns a placebo emancipation date using the main treatment and control groups and pre-emancipation data only.

A.7 County Patenting around Emancipation

Using inventor locations from Petralia et al. (2016), I sum county patent counts within the South (S) and comparison region (N) in each period. The South comprises the 1860 slave states, including Missouri and DC, with West Virginia classified alongside Virginia. The ratio S/N distinguishes Southern catch-up from growth in patenting shared by both regions. I calculate the aggregate relative change as

$$\Delta^{S/N} = 100 \left[\frac{S_{\text{post}}/N_{\text{post}}}{S_{\text{pre}}/N_{\text{pre}}} - 1 \right],$$

$$\Delta^{S/N}_{\text{full US, war included}} = 100 \left[\frac{121{,}693/820{,}335}{3{,}937/35{,}379} - 1 \right] = 33.3\%.$$

Table 13 reports totals, ratios and the corresponding Poisson estimates.

Table 13: County Patenting: Aggregate Changes and Poisson Estimates

A. Total patent counts

Sample / war years	South, pre	North, pre	South, post	North, post
Full US, war included	3,937	35,379	121,693	820,335
No West, war included	3,937	34,988	121,693	759,486
Full US, war excluded	3,157	22,499	121,367	814,942
No West, war excluded	3,157	22,335	121,367	754,171

B. Aggregate South/North ratios and relative changes

Sample	War years	Pre-period South/North	Post-period South/North	Relative change (%)
Full US	Included	0.1113	0.1483	+33.3
No West	Included	0.1125	0.1602	+42.4
Full US	Excluded	0.1403	0.1489	+6.1
No West	Excluded	0.1413	0.1609	+13.9

C. Poisson difference-in-differences

	1845–1864 baseline		1845–1860 baseline	
	Full US	No West	Full US	No West
	(1)	(2)	(3)	(4)
South × Post	0.287*	0.353**	0.060	0.130
County-clustered SE	(0.148)	(0.146)	(0.149)	(0.147)
County-clustered p-value	0.052	0.016	0.689	0.377
State-clustered SE	(0.250)	(0.244)	(0.264)	(0.257)
State-clustered p-value	0.250	0.147	0.822	0.614
Implied relative change (%)	+33.3	+42.4	+6.1	+13.9
Post-period	1865–1914	1865–1914	1866–1914	1866–1914
County and period fixed effects	Yes	Yes	Yes	Yes
Observations	43,092	37,212	43,078	37,198
Counties	3,078	2,658	3,077	2,657

Notes: Counts sum distinct patent IDs within counties; a patent linked to multiple counties may contribute more than once. North denotes non-Southern comparison counties. Full US includes the West; No West excludes the Census Mountain and Pacific divisions. With war years included, pre is 1845–1864 and post is 1865–1914; excluding 1861–1865, pre is 1845–1860 and post is 1866–1914. All endpoints are inclusive. In the latter sample, the 1860 bin retains only 1860 and the 1865 bin retains 1866–1869. Panel C uses Poisson QMLE with county and bin fixed effects, without a population offset. Its implied changes, $100[\exp(\beta) - 1]$, equal the aggregate changes in panel B in this balanced specification. Stars use county-clustered inference: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. State-clustered inference is reported separately. Pre-period differences and concurrent Southern shocks limit causal interpretation.

A.8 Emancipation and Farm Capital Shares

Adoption and the border comparison. Emancipation can strengthen incentives to mechanize before adoption becomes visible in the census as invention, diffusion, and investment take time. Comparing farms that grow similar crops helps distinguish the adoption of technologies suited to common tasks from differences in the technologies required by different crops. I therefore compare counties near the 1860 boundary between slave and free states, where farms on both sides could use similar grain-harvesting and cultivation equipment to see the immediate effect of the abolition. Figure 28 maps the cumulative 25, 50, and 100 km samples. Distances are measured to the nearest border county centroid. This geographic restriction improves the relevance of the comparison group, as shown below by comparing the crop mixes of counties.

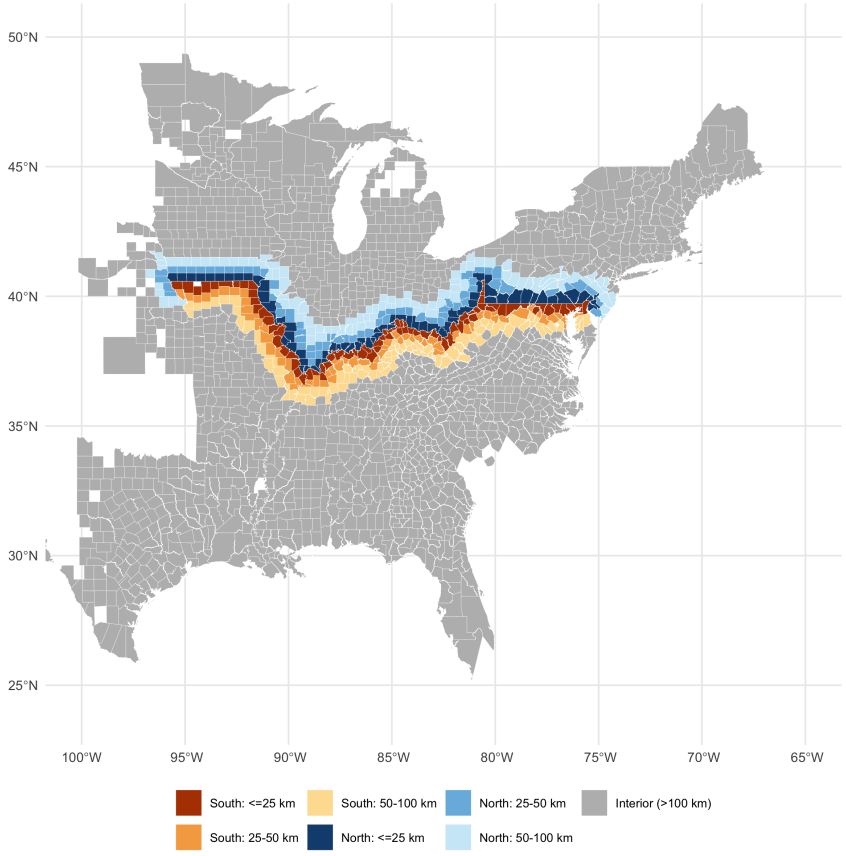


Figure 28: County Samples near the 1860 Slave–Free-State Border

Notes: Blue shades identify Northern counties and orange shades Southern counties. Shades distinguish distances of at most 25 km, 25–50 km, and 50–100 km to the nearest 1860 border county centroid. The regression samples use cumulative bands; gray identifies counties outside the 100 km corridor. Geographic coverage precedes outcome-specific sample restrictions.

Crop comparability. Figure 29 reports prewar South–North differences in product composition. Indeed, farms in counties within 25 km of the border have similar crop shares. However, as the distance increases, farms become less similar in terms of the products they produce. These comparisons motivate studying nearby farms facing a more similar set of agricultural tasks to check for an immediate increase in the capital share following the abolition of slavery.

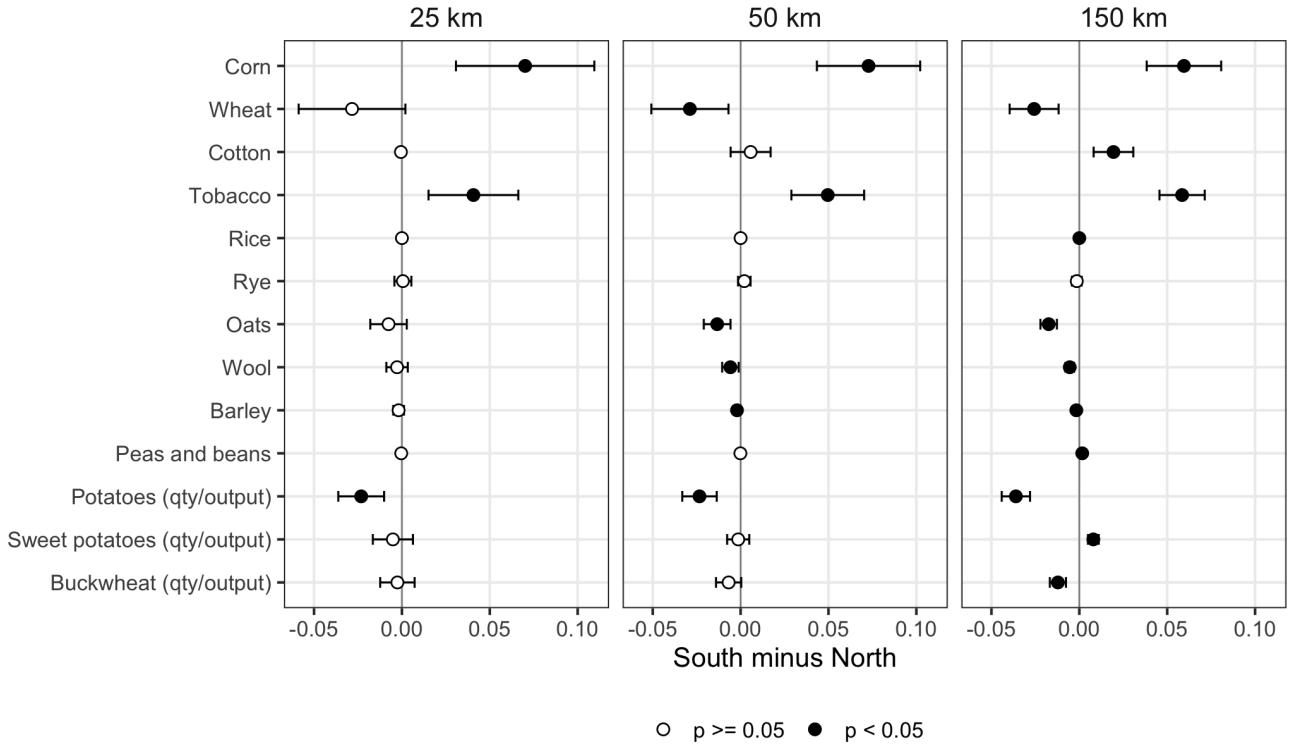


Figure 29: Prewar Product Composition across the Border, 1860

Notes: Points show differences in county means, South minus North, for cumulative 25, 50, and 150 km samples. Products are measured as value divided by agricultural output, except potatoes, sweet potatoes, and buckwheat, which are in terms of quantity/output ratios and are labeled accordingly. Bars are 95% confidence intervals; filled points indicate $p < 0.05$. The descriptive sample requires positive farm counts and agricultural output in 1860 and precedes the regression outcome restrictions.

Table 14: Descriptive Statistics of Farm and Manufacturing Samples, 1860

	Full sample	25 km	50 km	100 km
Panel A. County coverage				
Counties	1,954	148	276	456
South	1,113	77	139	222
North	841	71	137	234
Panel B. Agriculture				
Observations	1,871	148	276	456
Capital share (%): mean	12.20	13.23	13.04	13.16
Standard deviation	5.44	3.14	3.29	3.48
Output ($1,000): mean	856.40	977.67	925.35	940.17
Standard deviation	858.83	1,025.23	920.06	862.11
Buildings ($1,000): mean	659.79	955.72	874.42	888.00
Standard deviation	870.13	1,267.48	1,129.70	1,033.62
Equipment ($1,000): mean	127.20	152.63	140.97	142.18
Standard deviation	164.63	196.85	176.13	167.54
Livestock ($1,000): mean	559.14	622.56	613.52	645.22
Standard deviation	512.93	549.26	502.98	500.34
Product inventories ($1,000): mean	272.04	377.20	357.16	366.29
Standard deviation	276.87	389.18	348.14	331.16
Panel C. Manufacturing				
Observations	1,662	147	269	433
Capital / output: mean	0.698	0.608	0.647	0.635
Standard deviation	0.695	0.565	0.733	0.643
Output ($1,000): mean	1,069.84	2,360.72	1,594.69	1,237.03
Standard deviation	6,098.93	12,067.78	9,111.94	7,229.06
Capital ($1,000): mean	581.68	1,236.36	884.20	700.57
Standard deviation	2,922.67	6,337.50	4,878.54	3,884.64

Notes: Statistics describe the 1860 baseline. Counties have equal weight. Dollar values are in thousands of current dollars; outcomes are reported in levels. Panel A includes all counties in the geographic sample; Panels B and C use the eligible observations for each sector. The farm capital share includes buildings, equipment, living livestock, and fourteen-product inventories, excluding land. Manufacturing capital/output is a capital-intensity ratio. Samples exclude the District of Columbia and the West. Distance bands are cumulative distances to the nearest 1860 border county centroid.

Outcomes and sample. The county panel covers 1850–1900. The agricultural outcome a_{ct} is the capital share excluding land defined in Appendix A.1, based on imputed gross capital returns divided by census output. Manufacturing is measured by $m_{ct} = K^M_{ct}/Y^M_{ct}$, reported capital divided by output, a capital-intensity ratio. Table 14 reports baseline statistics in levels.

Main specifications. The DiD estimates the post-emancipation change in Southern farm capital shares relative to Northern counties:

$$\log a_{ct} = \alpha_c + \delta_t + \beta\big(\text{Post}_t \times \text{South}_c\big) + \varepsilon_{ct}. \tag{A-8-1}$$

Here c indexes counties and t census years; Post_t equals one in 1870–1900 and zero in 1850–1860, South_c identifies Southern counties, and α_c and δ_t are county and year fixed effects. Stacking agriculture and manufacturing gives the DDD specification:

$$\begin{aligned}\log z_{cst} = \alpha_{cs} + \delta_t + \theta\big(\text{Post}_t \times \text{South}_c\big) + \kappa\big(\text{Post}_t \times \text{Farm}_s\big) \\ + \beta^{DDD}\big(\text{Post}_t \times \text{South}_c \times \text{Farm}_s\big) + \varepsilon_{cst},\end{aligned} \tag{A-8-2}$$

where s indexes sectors, $z_{cst} = a_{ct}$ for agriculture and $z_{cst} = m_{ct}$ for manufacturing, Farm_s identifies agriculture, and α_{cs} are county-by-sector fixed effects. The coefficient β^{DDD} measures the South–North change in agriculture relative to manufacturing. Manufacturing in the same county provides a comparison for the war shock that affects that county as a whole. The sample excludes the District of Columbia and the West. I drop observations with negative stocks, output, or capital shares. I also retain manufacturing observations only if their capital-to-output ratio is positive and finite. The DDD sample retains counties with an eligible observation in 1860. Standard errors are clustered by county.

Border estimates. Southern farm capital shares rise relative to Northern shares within each border band following emancipation (Table 15, Panel A). At 25 km, the DiD coefficient is 0.1297 log points (SE 0.0379), equivalent to a 13.9% relative increase. The DDD coefficient in equation (A-8-2) is 0.2962 (SE 0.0890): a 34.5% increase in the South–North ratio of agricultural capital shares relative to manufacturing capital intensity. The DDD estimates decline to 0.2087 and 0.1549 at 50 and 100 km, respectively, but remain positive and significant at the 1% level. This shows that, following emancipation, Southern counties closest to the border with the North and thus growing similar crops increased their capital shares.

Table 15: Emancipation and Farm Capital Shares, 1850–1900

	Full sample (1)	25 km (2)	50 km (3)	100 km (4)
Panel A. Difference-in-differences				
Post × South	-0.3762***	0.1297***	0.0925***	0.0391*
	(0.0152)	(0.0379)	(0.0270)	(0.0228)
Observations	12,293	883	1,629	2,687
Counties	2,403	165	301	492
R^2	0.762	0.725	0.738	0.723
County fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Panel B. Triple differences relative to manufacturing				
Post × South × Farm	-0.2294***	0.2962***	0.2087***	0.1549***
	(0.0286)	(0.0890)	(0.0648)	(0.0501)
Observations	20,860	1,608	2,999	4,981
Counties	1,877	147	272	450
R^2	0.887	0.885	0.883	0.882
County × sector fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes

Notes: Each column reports a separate regression. The agricultural outcome is the log of the imputed farm capital share, including buildings, equipment, living livestock, and fourteen-product inventories, excluding land. Panel B stacks agriculture with log(manufacturing capital/manufacturing output) and includes Post × South and Post × Farm. Post equals one in 1870–1900; the pre-period is 1850–1860. The full sample excludes the District of Columbia and the West. Farm observations require complete nonnegative stocks, positive output and capital shares, and land-plus-stock-return shares at most one; manufacturing observations require a positive finite capital/output ratio. Panel B retains counties with an eligible observation in 1860. Distance bands refer to border county centroids. Parentheses report standard errors clustered by county. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Dynamics. Figure 30 replaces the post indicators in equations (A-8-1) and (A-8-2) with census-year interactions, using 1860 as the reference. DDD coefficients are positive in every postwar decade at each bandwidth. At 25 and 50 km, their 95% confidence intervals exclude zero from 1880 onward.

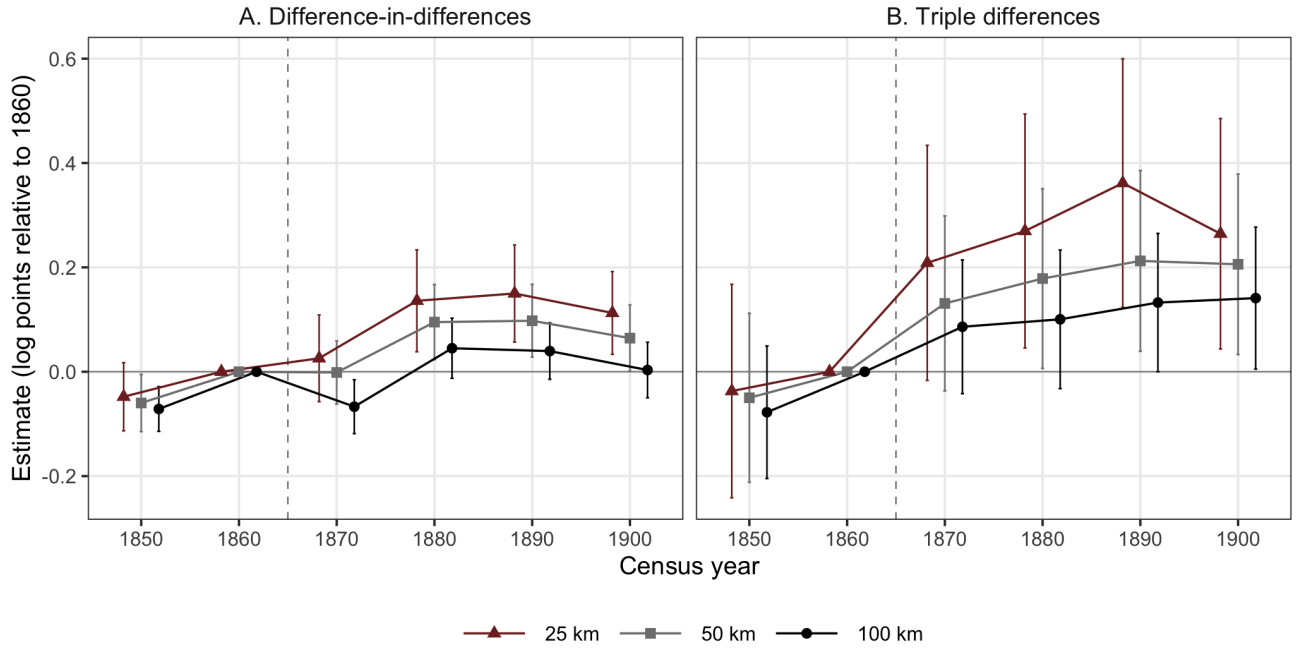


Figure 30: Farm Capital Shares around Emancipation: Border DiD and DDD Event Studies

Notes: Panel A reports year-by-South coefficients with county and year fixed effects. Panel B reports year-by-South-by-Farm coefficients with county-by-sector and year fixed effects and year-by-South and year-by-Farm interactions. Each series uses the same observations as its static specification in Table 15. Series are offset horizontally within each census year for readability. The reference year is 1860; the dashed vertical line marks emancipation in 1865. Intervals are county-clustered 95% confidence intervals. Bands are cumulative distances to border county centroids.

The full sample and wartime disruption. The nationwide comparison shows a pronounced Southern deterioration (Figure 31). The DiD coefficient falls to -0.443 in 1870 following the Civil War but recovers and increases after the initial shock until 1900; DDD coefficients remain negative throughout the postwar period. This pattern is consistent with a strong contribution from wartime destruction and financial disruption alongside emancipation (Feigenbaum et al., 2022; Goldin and Lewis, 1975). The initial decline recorded in 1870 is too large for the post-period coefficient to fully recover and become positive in the full U.S. sample. These patterns reinforce the rationale for comparing nearby farms, possibly with more similar wartime experiences, although farms on the Southern side of the border would still be expected to experience larger wartime shocks. The similarity of crops also gives these farms the possibility of adopting machinery immediately, compared with farms growing exclusively Southern crops.

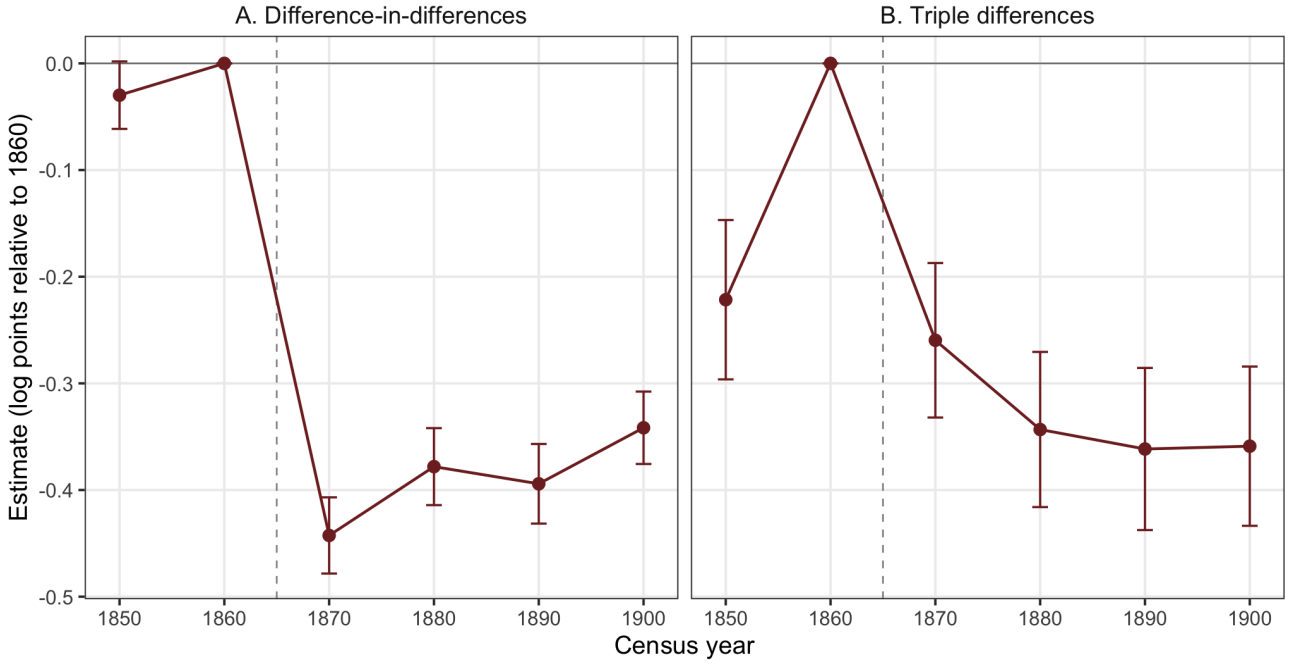


Figure 31: Farm Capital Shares around Emancipation: Full-Sample DiD and DDD Event Studies

Notes: Specifications match the border event studies in Figure 30, using the full estimation sample excluding the District of Columbia and the West. DiD compares Southern and Northern farm capital shares; DDD additionally differences against manufacturing capital intensity. The reference year is 1860. Intervals are county-clustered 95% confidence intervals.

B Model Derivations

B.1 Wages from Labor-Market Clearing

This appendix derives the wages in (12) for the task allocation in (9). The labor endowments are fixed at L and H. Low-skill labor performs tasks in $(I_t, \bar{I}_t]$; high-skill labor performs tasks in $(\bar{I}_t, 1]$ and produces the machine inputs used in $[0, I_t]$.

Low-skill wage. For a task assigned to low-skill labor, the technology and competitive task price imply

$$p_t(i)y_t(i) = \frac{w_{L,t}}{A_L c(i)\gamma(i)} A_L c(i)\gamma(i) l_t(i) = w_{L,t} l_t(i).$$

Equal task expenditure, (5), therefore gives $l_t(i) = Y_t / w_{L,t}$. Since low-skill employment is zero outside $(I_t, \bar{I}_t]$, its market-clearing condition is

$$\begin{aligned} L &= \int_{I_t}^{\bar{I}_t} l_t(i)\,\mathrm{d}i \\ &= \int_{I_t}^{\bar{I}_t} \frac{Y_t}{w_{L,t}}\,\mathrm{d}i = (\bar{I}_t - I_t)\frac{Y_t}{w_{L,t}}. \end{aligned}$$

Multiplying by $w_{L,t}$ and dividing by L yields

$$w_{L,t} = (\bar{I}_t - I_t)\frac{Y_t}{L}.$$

High-skill wage. In tasks performed directly by high-skill labor, the same argument gives $h_t(i) = Y_t / w_{H,t}$. Direct employment therefore equals $(1 - \bar{I}_t) Y_t / w_{H,t}$. High-skill labor is also needed to produce machines: each unit of $k_t(i)$ requires ψ units of labor. Using machine demand in (14), this employment is

$$\psi \int_0^{I_t} k_t(i)\,\mathrm{d}i = \psi \int_0^{I_t} \frac{\phi Y_t}{\psi w_{H,t}}\,\mathrm{d}i = \phi I_t \frac{Y_t}{w_{H,t}}.$$

The high-skill market-clearing condition combines both uses:

$$\begin{aligned} H &= \int_{\bar{I}_t}^{1} h_t(i)\,\mathrm{d}i + \psi \int_0^{I_t} k_t(i)\,\mathrm{d}i \\ &= (1 - \bar{I}_t)\frac{Y_t}{w_{H,t}} + \phi I_t \frac{Y_t}{w_{H,t}} \\ &= (1 - \bar{I}_t + \phi I_t)\frac{Y_t}{w_{H,t}}. \end{aligned}$$

Solving for the wage gives

$$w_{H,t} = (1 - \bar{I}_t + \phi I_t)\frac{Y_t}{H}. \tag{B-1-1}$$

The term $1 - \bar{I}_t$ reflects direct high-skill tasks, while ϕI_t reflects machine production. Combining the two wages yields

$$\frac{w_{L,t}}{w_{H,t}} = \frac{\bar{I}_t - I_t}{1 - \bar{I}_t + \phi I_t} \frac{H}{L}, \tag{B-1-2}$$

which is the wage ratio used in the equilibrium cutoff conditions (E1) and (E2).

B.2 Capital-Input Prices, Demand, and Profits

This appendix derives the capital-input price, demand in (14), and incumbent profits in (16). It first considers mechanized tasks whose limit price is set by the competitive fringe, then derives the profit of a newly mechanized task when labor constrains its price.

Limit price. Producing one unit of the capital input requires ψ units of high-skill labor, so its marginal cost is $\psi w_{H,t}$. The incumbent has quality $q_t(i)$, while competitive fringe firms have quality $\phi q_t(i)$. Their marginal costs per unit of task output are therefore

$$\underbrace{\frac{\psi w_{H,t}}{q_t(i)}}_{\text{Incumbent marginal cost}}, \qquad \underbrace{\frac{\psi w_{H,t}}{\phi q_t(i)}}_{\text{Fringe marginal cost}}.$$

The incumbent sets the task-output price equal to the marginal cost of the competitive fringe firms:

$$p_t(i) = \frac{\psi w_{H,t}}{\phi q_t(i)}, \tag{B-2-1}$$

recovering (13). The implied price per unit of capital input is $p_t(i)q_t(i) = \psi w_{H,t}/\phi$.

Demand and production. Equal task expenditure, (5), and $y_t(i) = q_t(i)k_t(i)$ imply

$$Y_t = p_t(i)y_t(i) = \frac{\psi w_{H,t}}{\phi q_t(i)} q_t(i)k_t(i) = \frac{\psi w_{H,t}}{\phi} k_t(i).$$

Solving for capital-input demand gives

$$k_t(i) = \frac{Y_t}{p_t(i)q_t(i)} = \frac{\phi Y_t}{\psi w_{H,t}}.$$

Production equals this demand. Each task therefore uses $\psi k_t(i) = \phi Y_t / w_{H,t}$ units of high-skill labor to produce its capital input. Substituting the wage in (B-1-1) also gives

$$k_t(i) = \frac{\phi H}{\psi(1 - \bar{I}_t + \phi I_t)}, \qquad y_t(i) = q_t(i) \frac{\phi H}{\psi(1 - \bar{I}_t + \phi I_t)}.$$

Incumbent profits. The incumbent's revenue is $p_t(i)y_t(i) = Y_t$, while its production cost is $\psi w_{H,t} k_t(i) = \phi Y_t$. Hence

$$\begin{aligned}\pi_t(i) &= p_t(i)y_t(i) - \psi w_{H,t} k_t(i) \\ &= Y_t - \psi w_{H,t} \frac{\phi Y_t}{\psi w_{H,t}} = (1-\phi) Y_t.\end{aligned}$$

Quality affects task output, but cancels from revenue and profits under fringe limit pricing.

Profits at the mechanization frontier. A newly mechanized task starts with quality $q_0(I_t)$. Its output price must also compete with the low-skill labor technology, so

$$p_t(I_t) = \min\left\{ \frac{\psi w_{H,t}}{\phi q_0(I_t)}, \frac{w_{L,t}}{A_L c(I_t)\gamma(I_t)} \right\}.$$

The switching point $\tilde{I}_t$ equates the two competing technologies' unit costs:

$$\frac{w_{L,t}}{A_L c(\tilde{I}_t)\gamma(\tilde{I}_t)} = \frac{\psi w_{H,t}}{\phi q_0(\tilde{I}_t)}.$$

Below this switching point, the fringe constrains the price; above it, labor constrains the price. The monotonic productivity schedules give the ordering used in (21).

Using $k_t(I_t) = Y_t / [p_t(I_t) q_0(I_t)]$, frontier profits are

$$\pi_t^m(I_t) = \left[1 - \frac{\psi w_{H,t}}{p_t(I_t) q_0(I_t)} \right] Y_t.$$

When the fringe sets the price, this expression equals $(1-\phi)Y_t$. When labor sets the price, it becomes

$$\pi_t^m(I_t) = \left[1 - \frac{\psi w_{H,t} A_L c(I_t)\gamma(I_t)}{w_{L,t} q_0(I_t)} \right] Y_t.$$

Together, these cases give (21). Profits reach zero at the stationary frontier, where the machine's unit production cost equals the unit cost of labor, as in (7).

B.3 Aggregation to Final Output

This appendix derives aggregate output in (15).[27] Splitting the task aggregator (2) according to the allocation in (9) gives

$$\begin{aligned}\ln Y_t = &\underbrace{\int_0^{I_t} [\ln q_t(i) + \ln k_t(i)]\,\mathrm{d}i}_{\text{Machines}} \\ &+ \underbrace{\int_{I_t}^{\bar{I}_t} [\ln(A_L c(i)\gamma(i)) + \ln l_t(i)]\,\mathrm{d}i}_{\text{Low-skill labor}} \\ &+ \underbrace{\int_{\bar{I}_t}^{1} [\ln(A_H \gamma(i)) + \ln h_t(i)]\,\mathrm{d}i}_{\text{High-skill labor}}.\end{aligned} \tag{B-3-1}$$

Substituting the wages derived in Appendix B.1 into (11) and (14) gives the input quantities in their respective task ranges:

$$l_t(i) = \frac{L}{\bar{I}_t - I_t}, \qquad h_t(i) = \frac{H}{1 - \bar{I}_t + \phi I_t}, \qquad k_t(i) = \frac{\phi}{\psi}\frac{H}{1 - \bar{I}_t + \phi I_t}.$$

The denominator $1 - \bar{I}_t + \phi I_t$ includes high-skill employment in both direct task production and machine production. Using these quantities in (B-3-1) and integrating the terms that are constant across tasks yields

$$\begin{aligned}\ln Y_t = &\int_0^{I_t} \ln q_t(i)\,\mathrm{d}i + I_t \ln\left(\frac{\phi}{\psi}\right) + I_t \ln\left(\frac{H}{1 - \bar{I}_t + \phi I_t}\right) \\ &+ \int_{I_t}^{\bar{I}_t} \ln[A_L c(i)\gamma(i)]\,\mathrm{d}i + (\bar{I}_t - I_t)\ln\left(\frac{L}{\bar{I}_t - I_t}\right) \\ &+ \int_{\bar{I}_t}^{1} \ln[A_H \gamma(i)]\,\mathrm{d}i + (1 - \bar{I}_t)\ln\left(\frac{H}{1 - \bar{I}_t + \phi I_t}\right).\end{aligned}$$

Collect the productivity terms and the machine-production factor in Q_t:

$$Q_t \equiv \exp\left\{\int_0^{I_t} \ln q_t(i)\,\mathrm{d}i + \int_{I_t}^{\bar{I}_t} \ln[A_L c(i)\gamma(i)]\,\mathrm{d}i + \int_{\bar{I}_t}^{1} \ln[A_H \gamma(i)]\,\mathrm{d}i\right\}\left(\frac{\phi}{\psi}\right)^{I_t}.$$

Combining the two terms involving $H/(1 - \bar{I}_t + \phi I_t)$ gives

$$\ln Y_t = \ln Q_t + (\bar{I}_t - I_t)\ln\left(\frac{L}{\bar{I}_t - I_t}\right) + (1 - \bar{I}_t + I_t)\ln\left(\frac{H}{1 - \bar{I}_t + \phi I_t}\right).$$

Exponentiating yields

$$Y_t = Q_t \left(\frac{L}{\bar{I}_t - I_t}\right)^{\bar{I}_t - I_t}\left(\frac{H}{1 - \bar{I}_t + \phi I_t}\right)^{1 - \bar{I}_t + I_t},$$

[27] Mechanization takes place sequentially at the marginal task I_t, while vertical innovation can improve any previously mechanized task. Mechanization costs are much higher than vertical innovation costs. The measure of tasks with labor-constrained prices is treated as negligible during the transition and vanishes asymptotically as mechanization ceases and quality improvements continue. For aggregation, established mechanized tasks are therefore priced according to (B-2-1).

which is (15). The exponent on H includes both the high-skill tasks and the mechanized tasks, since machines are produced using high-skill labor. The two labor exponents sum to one, so output exhibits constant returns to scale in L and H conditional on technology and the task allocation. On the balanced-growth path, setting $I_t = I^*$ and $\bar{I}_t = \bar{I}$ gives (24).

B.4 Proof of the Task-Allocation Lemma

This appendix proves Lemma 1.

Proof. To see this, consider the following intuitive argument, similar to Acemoglu and Autor (2011). Consider first the capital input relative to coerced low-skill labor. Their relative unit cost is

$$\frac{\psi w_H / q_0(i)}{w_L / [A_L c(i)\gamma(i)]} = \psi \frac{w_H}{w_L} \frac{A_L c(i)\gamma(i)}{q_0(i)}.$$

Since $c(i)\gamma(i)$ is strictly increasing and $q_0(i)$ is strictly decreasing, this ratio is strictly increasing in i. Therefore, if both inputs are used in an interior allocation, continuity implies that there exists a unique task I^* at which their unit costs are exactly equal. Since the relative unit cost is increasing in i, the capital input has a lower unit cost for tasks below I^*, whereas coerced low-skill labor has a lower unit cost for tasks above I^*. Intuitively, the capital input is relatively more productive at lower-indexed tasks because $q_0(i)$ is higher there.

Next consider coerced low-skill labor relative to high-skill labor:

$$\frac{w_L / [A_L c(i)\gamma(i)]}{w_H / [A_H \gamma(i)]} = \frac{w_L}{w_H} \frac{A_H}{A_L} \frac{1}{c(i)}.$$

Because $c(i)$ is strictly decreasing, this ratio is also strictly increasing in i. Hence, if both labor inputs are used, continuity implies that there exists a unique task $\bar{I}$ at which their unit costs are exactly equal. Coerced low-skill labor has a lower unit cost below $\bar{I}$, whereas high-skill labor has a lower unit cost above $\bar{I}$.

Under the interior ordering $I^* < \bar{I}$, these two comparisons are sufficient to determine the global assignment. For $i < I^*$, the capital input has a lower unit cost than coerced low-skill labor, which in turn has a lower unit cost than high-skill labor. For $I^* < i < \bar{I}$, coerced low-skill labor has a lower unit cost than both alternatives. For $i > \bar{I}$, high-skill labor has a lower unit cost than coerced low-skill labor, which in turn has a lower unit cost than the capital input. Thus, the allocation in (6) follows.

B.5 Derivation of the Equilibrium Cutoff Conditions

This appendix obtains (E1) and (E2) by combining cost indifference at the two task boundaries with the equilibrium wage ratio. Setting $I_t = I^*$ and $\bar{I}_t = \bar{I}$ in (B-1-2) gives

$$\frac{w_L}{w_H} = \frac{\bar{I} - I^*}{1 - \bar{I} + \phi I^*} \frac{H}{L}. \tag{B-5-1}$$

Mechanization frontier: E1. At I^*, a new machine with quality $q_0(I^*)$ earns zero profits when its marginal production cost equals the cost of performing the task with low-skill labor. Rearranging (7) gives

$$\frac{\psi w_H}{q_0(I^*)} = \frac{w_L}{A_L c(I^*)\gamma(I^*)} \quad \Longrightarrow \quad \frac{w_L}{w_H} q_0(I^*) = \psi A_L c(I^*)\gamma(I^*).$$

Substituting (B-5-1) yields (E1):

$$\frac{\bar{I} - I^*}{1 - \bar{I} + \phi I^*} \frac{H}{L} q_0(I^*) = \psi A_L \gamma(I^*) c(I^*).$$

The frontier condition uses the initial machine quality and its marginal production cost, since it concerns the profitability of further mechanization.

Labor cutoff: E2. At $\bar{I}$, low- and high-skill labor have equal task-production costs. Canceling their common task-productivity term $\gamma(\bar{I})$ in (8) gives

$$\frac{w_L}{A_L c(\bar{I})\gamma(\bar{I})} = \frac{w_H}{A_H \gamma(\bar{I})} \quad \Longrightarrow \quad \frac{w_L}{w_H} A_H = A_L c(\bar{I}).$$

Substituting the same wage ratio yields (E2):

$$\frac{\bar{I} - I^*}{1 - \bar{I} + \phi I^*} \frac{H}{L} A_H = A_L c(\bar{I}).$$

Together, the two conditions determine the stationary cutoffs $(I^*, \bar{I})$ for given labor endowments and productivity schedules.

B.6 Proof of the Balanced-Growth Rate

Along the balanced-growth path, task allocation is stationary. Equation (24) therefore depends on time only through aggregate productivity Q_t, implying

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{Q}_t}{Q_t}.$$

Vertical innovation arrives at rate τ^* on each mechanized task and raises machine quality by the factor $1 + \lambda$. Thus, aggregate log productivity evolves according to

$$\begin{aligned} \ln Q(t + \Delta t) - \ln Q(t) &= \int_0^{I^*} \tau^* \Delta t \ln(1 + \lambda)\, di \\ &= I^* \tau^* \Delta t \ln(1 + \lambda). \end{aligned}$$

Dividing by Δt and taking the limit as $\Delta t \to 0$ gives the balanced-growth rate:

$$g^* = \frac{\dot{Q}_t}{Q_t} = \lim_{\Delta t \to 0} \frac{\ln Q(t + \Delta t) - \ln Q(t)}{\Delta t} = I^* \tau^* \ln(1 + \lambda).$$

■

C Proof of Proposition 1

Dividing (E1) by (E2) yields the key equilibrium condition

$$c(\bar{I}) = \underbrace{\frac{\psi A_H \gamma(I^*) c(I^*)}{q_0(I^*)}}_{\text{strictly increasing in } I^*}. \tag{C-0-1}$$

I first establish uniqueness conditional on existence. Equation (E2) defines a unique and strictly increasing relationship between $\bar{I}$ and I^*. Intuitively, for a given I^*, the left-hand side in (E2) is strictly increasing in $\bar{I}$, while $c(\bar{I})$ on the right-hand side is strictly decreasing. Moreover, an increase in I^* lowers the left-hand side for a given $\bar{I}$, so the equality can be restored only through an increase in $\bar{I}$. Hence $\bar{I} = \bar{I}(I^*)$ is strictly increasing.

Since $c(\cdot)$ is strictly decreasing, it follows that

$$\underbrace{c(\bar{I}(I^*))}_{\text{strictly decreasing in } I^*}$$

is strictly decreasing in I^*. By assumption, $\gamma(I)c(I)$ is strictly increasing, while $q_0(I)$ is strictly decreasing. Therefore

$$\underbrace{\frac{\psi A_H \, \gamma(I^*) \, c(I^*)}{q_0(I^*)}}_{\text{strictly increasing in } I^*}$$

is strictly increasing. Equation (C-0-1) thus equates a strictly decreasing function with a strictly increasing function. Consequently, if an equilibrium exists, it is unique.

We next establish existence. Let $I^*_{\max}$ denote the largest admissible value of I^* consistent with $\bar{I}, I^* \in [0,1]$ when, $\bar{I} = 1$.[28] Hence, evaluating (C-0-1) at $I^*_{\max}$ gives

$$c(1) = \frac{\psi A_H \, \gamma(I^*_{\max}) \, c(I^*_{\max})}{q_0(I^*_{\max})}.$$

Under the condition in Proposition 1, we have

$$\frac{\psi A_H \, \gamma(I^*_{\max})}{q_0(I^*_{\max})} \geq 1.$$

It follows that

$$\frac{\psi A_H \, \gamma(I^*_{\max}) \, c(I^*_{\max})}{q_0(I^*_{\max})} \geq c(I^*_{\max}).$$

Since $I^*_{\max} < 1$ and $c(\cdot)$ is strictly decreasing,

$$c(I^*_{\max}) > c(1).$$

[28] Evaluating (E2) at $\bar{I} = 1$ gives $I^*_{\max} = \frac{1}{1+\phi \frac{A_L L}{A_H H} c(1)}$.

Therefore, at the upper admissible cutoff,

$$\frac{\psi A_H \, \gamma(I^*_{\max}) \, c(I^*_{\max})}{q_0(I^*_{\max})} > c(1).$$

At the opposite boundary, as $I^* \to 0^+$, $q_0(I^*) \to \infty$. Hence

$$\frac{\psi A_H \, \gamma(I^*) \, c(I^*)}{q_0(I^*)} \longrightarrow 0,$$

whereas $c(\bar{I}) > 0$. Thus the left-hand side of (C-0-1) lies above the right-hand side near the lower boundary, while the right-hand side lies above the left-hand side at the upper boundary. By continuity, the two sides must cross at least once.

Since the crossing is unique by the argument above, the equilibrium $(\bar{I}, I^*)$ exists and is unique. □

D Two Region Extension Derivations

D.1 Regional Equilibrium and Separability

This appendix derives the regional counterparts of the equilibrium conditions (E1) and (E2). Let $f \in \{N, S\}$ index regions, $E^f_t = P^f_t Y^f_t$ denote regional expenditure, and $c^f(i)$ denote the regional coercion schedule. Labor and asset markets are segmented as in Section 4.1.

Regional households. The representative household in region $f \in \{N, S\}$ has the same preferences as in the one-region model and solves

$$U^f(t) = \int_t^\infty \exp\left(-\rho(s-t)\right) \ln C^f(s) \, ds,$$

subject to

$$\dot{\mathcal{A}}^f_t + C^f_t \leq r^f_t \mathcal{A}^f_t + w^f_{H,t} H^f + w^f_{L,t} L^f.$$

Here, $\mathcal{A}^f_t$ is the balanced portfolio of innovating firms located in region f, with

$$\mathcal{A}^f_t = \int_0^{I^f_t} V^f_t(i) \, di.$$

Households consume the national final good and save only in regional assets. With segmented asset markets, regional rates of return need not be equalized. The household first-order condition gives

$$\frac{\dot{C}^f_t}{C^f_t} = r^f_t - \rho.$$

Regional expenditure and factor prices. The first-order conditions for the national CES aggregator (32) give

$$P^f_t = \chi_f^{1/\varepsilon} \left(\frac{Y_t}{Y^f_t}\right)^{1/\varepsilon}, \qquad \chi_N = \chi, \quad \chi_S = 1 - \chi.$$

With the national final-good price normalized to one, zero profits imply $Y_t = E_t^N + E_t^S$. Within each region, the Cobb–Douglas task aggregator implies equal expenditure across tasks:

$$p_t^f(i) y_t^f(i) = E_t^f.$$

Using the labor unit costs and machine limit price derived in the main text, input demands are $l_t^f(i) = E_t^f / w_{L,t}^f$ in low-skill tasks, $h_t^f(i) = E_t^f / w_{H,t}^f$ in high-skill tasks, and $k_t^f(i) = \phi E_t^f / (\psi w_{H,t}^f)$ in mechanized tasks. Thus, for the allocation $[0, I_t^f]$, $(I_t^f, \bar{I}_t^f]$, and $(\bar{I}_t^f, 1]$, labor-market clearing gives

$$L^f = (\bar{I}_t^f - I_t^f) \frac{E_t^f}{w_{L,t}^f},$$
$$H^f = (1 - \bar{I}_t^f + \phi I_t^f) \frac{E_t^f}{w_{H,t}^f}.$$

The term ϕI_t^f accounts for high-skill labor used to produce machines. It follows that

$$w_{L,t}^f = (\bar{I}_t^f - I_t^f) \frac{E_t^f}{L^f}, \qquad w_{H,t}^f = (1 - \bar{I}_t^f + \phi I_t^f) \frac{E_t^f}{H^f}. \tag{D-1-1}$$

Both wages scale with E_t^f, so regional expenditure cancels from their ratio and hence from the task-cutoff conditions.

Stationary equilibrium cutoffs. Let $I^{f,\star}$ be the stationary mechanization frontier and $\bar{I}^f$ the corresponding labor cutoff. At the frontier, a new machine with initial quality $q_0(I^{f,\star})$ earns zero profits when its production cost equals the unit cost of low-skill labor. At the labor cutoff, low- and high-skill unit costs are equal. These two conditions are

$$\frac{\psi w_H^f}{q_0(I^{f,\star})} = \frac{w_L^f}{A_L c^f(I^{f,\star}) \gamma(I^{f,\star})},$$
$$\frac{w_L^f}{A_L c^f(\bar{I}^f) \gamma(\bar{I}^f)} = \frac{w_H^f}{A_H \gamma(\bar{I}^f)}.$$

The first equality concerns the profitability of a newly introduced machine, so it uses marginal cost rather than the incumbent markup price. Substituting the wage ratio from (D-1-1) yields

$$\frac{\bar{I}^f - I^{f,\star}}{1 - \bar{I}^f + \phi I^{f,\star}} \frac{H^f}{L^f} q_0(I^{f,\star}) = \psi A_L \gamma(I^{f,\star}) c^f(I^{f,\star}), \tag{E1^f}$$

$$\frac{\bar{I}^f - I^{f,\star}}{1 - \bar{I}^f + \phi I^{f,\star}} \frac{H^f}{L^f} A_H = A_L c^f(\bar{I}^f). \tag{E2^f}$$

These equations determine $(I^{f,\star}, \bar{I}^f)$ for given regional endowments and institutions. Dividing (E1^f) by (E2^f) gives the regional mechanization condition

$$\frac{q_0(I^{f,\star})}{\gamma(I^{f,\star})} = \psi A_H \frac{c^f(I^{f,\star})}{c^f(\bar{I}^f)}.$$

In the North, $c^N(i) = 1$, so the frontier satisfies $q_0(I^{N,\star})/\gamma(I^{N,\star}) = \psi A_H$. Before abolition, $c^S(i) = c(i) > 1$. Under the decreasing coercion schedule and $\bar{I}^S > I^{S,\star}$, the ratio $c(I^{S,\star})/c(\bar{I}^S)$ exceeds one. Since $q_0(i)/\gamma(i)$ is strictly decreasing, $I^{S,\star} < I^{N,\star}$, corresponding to $I^* < I^{**}$ in the main text.

Completing the regional equilibrium. Once the cutoffs are determined, regional innovation and growth follow the conditions in Section 4.2. Profits and R&D costs both scale with E_t^f, so the normalized innovation conditions retain their one-region form. The CES system determines regional prices and expenditure levels; segmented factor markets allow wages and asset returns to adjust within each region. At abolition, $c^S(i)$ becomes one, changing the South's stationary target. Its approach to that target is governed by (33).

E Quantitative Details

E.1 Details of the Quantitative Implementation

This appendix provides the equations used to implement the four-step quantitative procedure described in Section 4.3.3. Throughout, the candidate parameter vector is

$$\Theta = (I^{**}, I^*, \tau^*, \xi, I_{N,1800}, I_{S,1800}, \bar{I}, \beta_c), \qquad \xi \equiv \frac{\kappa^m}{\kappa}.$$

Here, $\bar{I}$ denotes the stationary labor-allocation cutoff in the coercive economy.

Step 1: Stationary equilibria and implied parameters. Recall the functional forms

$$\gamma(i) = e^{\beta i}, \qquad c(i) = Ce^{-\beta_c i}, \qquad q_0(i) = \frac{\bar{q}e^{-\beta_q i}}{i},$$

and define

$$a \equiv \beta + \beta_q.$$

The stationary low- versus high-skill labor cutoff in the coercive economy satisfies

$$\frac{\bar{I} - I^*}{1 - \bar{I} + \phi I^*} \frac{H^S}{L^S} A_H = A_L c(\bar{I}).$$

Hence,

$$c(\bar{I}) = \frac{\bar{I} - I^*}{1 - \bar{I} + \phi I^*} \frac{H^S}{L^S} \frac{A_H}{A_L}.$$

Since $c(\bar{I}) = Ce^{-\beta_c \bar{I}}$, the level of the coercion technology is

$$C = c(\bar{I})e^{\beta_c \bar{I}}. \tag{E-1-1}$$

The stationary mechanization condition under free labor is

$$\frac{q_0(I^{**})}{\gamma(I^{**})} = \psi A_H, \tag{E-1-2}$$

while under coercion it is

$$\frac{q_0(I^{*})}{\gamma(I^{*})} = \psi A_H \frac{c(I^{*})}{c(\bar{I})}. \tag{E-1-3}$$

Using the functional forms above,

$$\frac{q_0(i)}{\gamma(i)} = \frac{\bar{q}e^{-ai}}{i}.$$

Combining (E-1-2) and (E-1-3) therefore gives

$$a = \frac{\ln\left[\dfrac{c(I^{*})}{c(\bar{I})}\dfrac{I^{*}}{I^{**}}\right]}{I^{**} - I^{*}}. \tag{E-1-4}$$

The free-labor stationary condition then determines the relative technology scale,

$$\frac{\psi}{\bar{q}} = \frac{e^{-aI^{**}}}{A_H I^{**}}. \tag{E-1-5}$$

The stationary vertical innovation intensity τ^* is estimated directly as an element of Θ. Conditional on τ^*, the stationary innovation condition

$$\rho + \tau^* = \frac{1-\phi}{\alpha\kappa(\tau^*)^{\alpha-1}}$$

determines

$$\kappa = \frac{1-\phi}{\alpha(\tau^*)^{\alpha-1}(\rho+\tau^*)}. \tag{E-1-6}$$

Finally, the candidate R&D cost ratio ξ determines

$$\kappa^m = \xi\kappa. \tag{E-1-7}$$

Thus, conditional on Θ, equations (E-1-1), (E-1-4), (E-1-5), (E-1-6), and (E-1-7) recover the remaining parameters required to solve the transition.

The implied long-run Northern growth rate is not used to recover τ^*. Instead, it provides an estimation moment:

$$g_N^{LR}(\Theta) = I^{**}\tau^* \ln(1+\lambda).$$

Step 2: Stable transition paths. The transition equations have the same form in both regions, conditional on whether coercion is available. I therefore suppress regional superscripts in this step and use

F and C to distinguish the free-labor and coercive regimes.

Under free labor, the current profitability of mechanizing the marginal task relative to vertical innovation is

$$\frac{\pi^{m,F}(I)}{\pi^{F}(I)} = \frac{1 - A_H \dfrac{\psi}{\bar{q}} I e^{aI}}{1-\phi}.$$

Under coercion, I write the contemporaneous labor-allocation cutoff as $\bar{I}(I)$, using the same cutoff notation as in the main model. The corresponding relative profitability is

$$\frac{\pi^{m,C}(I)}{\pi^{C}(I)} = \frac{1 - A_L \dfrac{\psi}{\bar{q}} c(I) I e^{aI} \dfrac{1 - \bar{I}(I) + \phi I}{\bar{I}(I) - I} \dfrac{L^S}{H^S}}{1-\phi}.$$

The stationary conditions imply

$$\frac{\pi^{m,F}(I^{**})}{\pi^{F}(I^{**})} = 0, \qquad \frac{\pi^{m,C}(I^{*})}{\pi^{C}(I^{*})} = 0, \qquad \bar{I}(I^{*}) = \bar{I}.$$

Using (33) and the estimated cost ratio $\xi = \kappa^m/\kappa$, mechanization and the relative value of mechanization evolve according to

$$\begin{aligned} \dot{I} &= \tau \left(\frac{v}{\xi} \right)^{\frac{1}{\alpha-1}}, \\ \dot{v} &= \frac{1-\phi}{\alpha \kappa \tau^{\alpha-1}} \left(v - \frac{\pi^m(I)}{\pi(I)} \right). \end{aligned} \tag{E-1-8}$$

The profit ratio in (E-1-8) is given by the free-labor or coercive expression above, depending on the regime.

Substituting the expression for $\dot{I}$ into the resource constraint gives

$$\frac{C_t}{E_t} = 1 - \kappa \tau^{\alpha} \left(I + \xi^{-\frac{1}{\alpha-1}} v^{\frac{\alpha}{\alpha-1}} \right), \tag{E-1-9}$$

where E_t is regional income. The law of motion for vertical innovation is given by (33). I keep the consumption share C_t/E_t in the expression. Differentiating (E-1-9), using the equation for $\dot{I}$, and collecting the terms in $\dot{\tau}$ gives

$$\dot{\tau} = \frac{\tau \dfrac{C_t}{E_t} \left[\rho + \tau - \dfrac{1-\phi}{\alpha \kappa \tau^{\alpha-1}} \right] - \kappa \tau^{\alpha} \dot{I} \left[\tau + \dfrac{\alpha}{\alpha - 1} \dot{v} \right]}{\alpha - \dfrac{C_t}{E_t}}.$$

Here, $\dot{\tau}$ is the change in vertical innovation intensity over time. The consumption share is determined by (E-1-9), while $\dot{I}$ and $\dot{v}$ are given by the transition equations above.

The free-labor stable arm is the path

$$\left(I, \tau^F(I), v^F(I) \right),$$

satisfying

$$\tau^F(I) \to \tau^*, \qquad v^F(I) \to 0 \qquad \text{as } I \to I^{**}.$$

The coercive stable arm also includes the labor-allocation cutoff:

$$\left(I, \tau^C(I), v^C(I), \bar{I}(I)\right),$$

with

$$\tau^C(I) \to \tau^*, \qquad v^C(I) \to 0, \qquad \bar{I}(I) \to \bar{I} \qquad \text{as } I \to I^*.$$

The coercive labor cutoff is determined at every point on the transition by

$$\frac{\bar{I}(I) - I}{1 - \bar{I}(I) + \phi I} \frac{H^S}{L^S} A_H = A_L c\big(\bar{I}(I)\big). \tag{E-1-10}$$

Differentiating (E-1-10) with respect to I gives

$$\frac{d\bar{I}(I)}{dI} = \frac{\dfrac{H^S}{L^S} A_H \dfrac{1 - (1-\phi)\bar{I}(I)}{[1 - \bar{I}(I) + \phi I]^2}}{\dfrac{H^S}{L^S} A_H \dfrac{1 - (1-\phi)I}{[1 - \bar{I}(I) + \phi I]^2} + A_L \beta_c c\big(\bar{I}(I)\big)}. \tag{E-1-11}$$

For numerical convenience, I solve the stable arms using I as the independent variable:

$$\frac{d\tau}{dI} = \frac{\dot{\tau}}{\dot{I}}, \qquad \frac{dv}{dI} = \frac{\dot{v}}{\dot{I}},$$

together with (E-1-11) under coercion.

Calendar time is recovered from

$$\frac{dt}{dI} = \frac{1}{\tau(I)} \left(\frac{\xi}{v(I)}\right)^{\frac{1}{\alpha - 1}}.$$

Thus, starting from I_0,

$$t - t_0 = \int_{I_0}^{I_t} \frac{1}{\tau(I)} \left(\frac{\xi}{v(I)}\right)^{\frac{1}{\alpha - 1}} dI. \tag{E-1-12}$$

Local initialization of the stable arms. The transition cannot be initialized exactly at the stationary frontier because $v = 0$ and hence $\dot{I} = 0$. The numerical solution is therefore initialized a small distance below the stationary frontier. See the Online Appendix for the local approximation and numerical initialization details.

Step 3: Historical regional transitions. The estimated initial mechanization levels locate each region on the appropriate stable arm. In 1800,

$$\tau_{N,1800} = \tau^F(I_{N,1800}), \qquad v_{N,1800} = v^F(I_{N,1800}),$$
$$\tau_{S,1800} = \tau^C(I_{S,1800}), \qquad v_{S,1800} = v^C(I_{S,1800}), \qquad \bar{I}_{S,1800} = \bar{I}(I_{S,1800}).$$

The calendar-time mapping in (E-1-12) is then used to advance each region from 1800 to subsequent dates.

Until 1865, the North follows the free-labor arm and the South follows the coercive arm. At abolition, mechanization is predetermined:

$$I_{S,1865^+} = I_{S,1865^-}.$$

The forward-looking variables jump to the free-labor stable arm:

$$\tau_{S,1865^+} = \tau^F(I_{S,1865}), \qquad v_{S,1865^+} = v^F(I_{S,1865}).$$

The South subsequently follows the free-labor arm toward I^{**}.

The postwar change in the Southern labor endowment ratio is imposed at abolition and is used when computing the postwar static labor allocation and regional output.

Step 4: Model moments. The stable paths determine I_t^N and I_t^S at each observation date. The CES demand system implies

$$\frac{E_t^S}{E_t^N} = \frac{E_{1800}^S}{E_{1800}^N}\left[\frac{Y_t^S/Y_t^N}{Y_{1800}^S/Y_{1800}^N}\right]^{\frac{\varepsilon-1}{\varepsilon}}.$$

The Southern expenditure share is therefore

$$\omega_t^S = \frac{E_t^S/E_t^N}{1+E_t^S/E_t^N}.$$

The model-implied national capital income share is

$$\mathcal{K}_t = \phi\left[(1-\omega_t^S)I_t^N + \omega_t^S I_t^S\right].$$

The long-run Northern growth moment is

$$g_N^{LR}(\Theta) = I^{**}\tau^* \ln(1+\lambda).$$

Let $I_{65} \equiv I_{S,1865^-}$ and $b_{65} \equiv \bar{I}_{S,1865^-}$. The model-implied high- relative to low-skill hire-cost ratio immediately before abolition is

$$\frac{w_{H,65}^S}{w_{L,65}^S} = \frac{1-b_{65}+\phi I_{65}}{b_{65}-I_{65}}\frac{L^S}{H^S}.$$

For the plantation-productivity restriction, define the share-equivalent free-labor cutoff at the abolition allocation as

$$b_{65}^F = I_{65} + \frac{\dfrac{A_L}{A_H}\dfrac{L^S}{H^S}}{1+\dfrac{A_L}{A_H}\dfrac{L^S}{H^S}}\left[1-(1-\phi)I_{65}\right].$$

The change in log productivity associated with removing coercion is

$$\Delta \ln Q \equiv \ln Q^F - \ln Q^C = (b_{65}^F - b_{65}) \ln \left(\frac{A_L}{A_H} \right) - \int_{I_{65}}^{b_{65}} \ln c(i)\, di.$$

Given $c(i) = Ce^{-\beta_c i}$,

$$\int_{I_{65}}^{b_{65}} \ln c(i)\, di = (b_{65} - I_{65}) \ln C - \frac{\beta_c}{2} \left(b_{65}^2 - I_{65}^2 \right).$$

Hence the model-implied coercive-to-free productivity ratio is

$$\frac{Q^C}{Q^F} = e^{-\Delta \ln Q}.$$

Together with the model-implied relative North–South GDP-per-worker growth rate, these objects form the empirical counterparts used in the minimum-distance estimation.

E.2 Initial Regional Capital Stocks around 1800

For the calibration I need the initial relative capital stock of the North and South around 1800. Since complete state-level estimates are not available for this date, I combine national capital-stock estimates with historical evidence on how the different components of capital were distributed across regions. I use reproducible domestic capital, excluding slave wealth and unimproved land, to match the capital concept in the model.

I start with the 1799 estimates of Gallman and Rhode (2022). Their variant-B measure of domestic reproducible capital is approximately \$613 million in 1860 prices. It consists of \$206.0 million in structures, \$138.6 million in animals, \$32.7 million in shipping, \$20.0 million in other equipment, and \$215.0 million in inventories.

I then allocate each component between the North and South using the available historical evidence. For structures, I use the distribution of houses and lands reported by Parker (1848), based on the 1798 Direct Tax. Across the eight Northern and eight Southern states included in the analysis, the North accounted for approximately 68.1 percent of these values. I also use this share to distribute the equipment value across states because a state-level allocation is not available. For shipping, I use the state distribution of registered foreign-trade tonnage in 1799 from North (1961). I also use this share for imported inventories, since shipping activity is closely related to these goods.

For animals and domestically produced inventories, I use the regional wealth estimates of Jones (1980), together with the regional population totals from Perkins (1988). These data imply Northern shares of approximately 47.4 percent for livestock and 46.9 percent for agricultural inventories. For mined and manufactured inventories, I use Jones's nonfarm business inventory category, which gives a Northern share of approximately 87.6 percent. This category does not correspond exactly to mined and manufactured inventories, so I also report a more conservative allocation below.

Combining these allocations gives

$$\frac{K_{N,1800}}{K_{S,1800}} = \frac{373.568}{238.732} = 1.565.$$

Thus, the North held approximately one and a half times as much reproducible capital as the South around 1800.

Table 16 compares this estimate with alternative calculations. A higher ratio of 2.178 is obtained when only structures, shipping, and equipment are included. Applying the 1798 property distribution to almost the entire Gallman–Rhode capital stock also gives a higher ratio of 2.153. However, this assumes that livestock and agricultural inventories were distributed across regions in the same way as real estate. The historical evidence instead suggests that these components were distributed much more evenly between the North and South.

Table 16: Alternative estimates of the North–South capital-stock ratio around 1800

Capital-stock measure	North	South	K_N/K_S
	millions of 1860 dollars		
Narrow structures, shipping, and equipment	177.3	81.4	2.178
Full stock, mechanical property allocation	418.1	194.2	2.153
Full stock, component-specific baseline	**373.6**	**238.7**	**1.565**
Conservative manufacturing-inventory allocation	362.6	249.7	1.450
Lindert–Williamson realty sensitivity	359.5	252.8	1.423
Broader capital including land improvement	589.1	509.2	1.16

Notes: The baseline includes reproducible domestic capital and excludes slave wealth. National component totals are from Gallman and Rhode (2022). Structures and other equipment are allocated using the 1798 property distribution reported by Parker (1848). Shipping and imported inventories use 1799 registered tonnage from North (1961). Animals and domestic inventories use regional wealth components from Jones (1980), population-weighted using Perkins (1988). The Lindert–Williamson calculation uses their alternative evidence on regional real-property values; its regional coverage does not exactly match the states used here. The final row also includes clearing and breaking of land and therefore uses a broader capital measure than the baseline.

I therefore use $K_{N,1800}/K_{S,1800} = 1.565$ as the calibration target and describe it in the main text as approximately 1.5. This calculation includes all the reproducible capital components listed above and uses separate regional information for each component where possible. The alternative full-stock allocations give ratios of approximately 1.4–1.6.

E.3 Sensitivity Analysis

Table 17 reports sensitivity to ε under the current fixed-labor specification. For each value, I jointly re-estimate all eight parameters using the same nine moments, including the 1800 national capital share and North–South capital-stock ratio. The historical GDP-per-worker moment is reconstructed using the corresponding elasticity, and the objective uses the same normalized moment discrepancies as the benchmark estimation.

Table 17: Sensitivity to the Regional Elasticity of Substitution

ε	I^{**}	I^{*}	I^{*}/I^{**}	$I_{N,1800}$	$I_{S,1800}$	τ^{*}	ξ	g_N^{LR} (%/yr)	$10^5 J$	1910 S/N GDP per worker Relative gap (%)
2	0.3737	0.1261	0.3374	0.1890	0.1261	0.4158	60.38	1.637	24661.290	86.14
3	0.3603	0.1273	0.3534	0.1938	0.1273	0.3522	66.71	1.337	2460.977	16.30
4	0.3695	0.1318	0.3567	0.1941	0.1277	0.3182	82.43	1.239	213.881	-0.63
4.25	0.3663	0.1412	0.3855	0.1940	0.1276	0.3204	86.90	1.236	149.161	-1.26
4.5	0.3631	0.1494	0.4115	0.1939	0.1276	0.3227	90.77	1.235	105.992	-1.67
4.75	0.3603	0.1567	0.4350	0.1938	0.1275	0.3250	94.27	1.234	75.722	-1.99
5	**0.3576**	**0.1632**	**0.4565**	**0.1937**	**0.1275**	**0.3272**	**97.40**	**1.233**	**53.997**	**-2.20**
6	0.3489	0.1833	0.5253	0.1935	0.1274	0.3349	107.56	1.231	12.744	-2.68
7	0.3403	0.1985	0.5833	0.1934	0.1274	0.3431	112.32	1.230	3.534	-2.21
8	0.3328	0.2097	0.6300	0.1933	0.1274	0.3508	112.39	1.230	3.003	-1.61
9	0.3297	0.2163	0.6560	0.1933	0.1274	0.3539	115.58	1.229	6.775	-1.93
10	0.3441	0.2135	0.6205	0.1933	0.1273	0.3392	146.74	1.230	11.562	-6.52

Notes: For each value of ε, all eight estimated parameters are jointly re-estimated using the current fixed-labor specification and the same nine moments. The historical GDP-per-worker moment is reconstructed using each elasticity. The prewar labor ratio is $L/H = 1.04$ and the postwar ratio is 0.7072. The benchmark $\varepsilon = 5$ is shown in bold. I^{**} and I^{*} denote the stationary free-labor and coercive mechanization frontiers. g_N^{LR} is the long-run Northern growth rate. $J = \sum_j r_j^2$ uses symmetric normalized residuals, $r_j = (m_j - d_j)/[(|m_j| + |d_j|)/2]$, as in the benchmark estimation; lower values indicate a better fit. The final column reports $100 \times (\text{model}/\text{data} - 1)$ for the South–North GDP-per-worker ratio in 1910, higher positive or negative values indicate greater distance to the actual data.

For all tested values with $\varepsilon \geq 4$, every targeted moment is within 3.6 percent of its empirical counterpart, and the implied long-run Northern growth rate lies between 1.229 and 1.239 percent per year, close to the target of 1.23 percent. For $4 \leq \varepsilon \leq 9$, the model also matches the untargeted South–North GDP-per-worker ratio in 1910 within 2.7 percent of its empirical counterpart. There is a unique interior equilibrium across the entire epsilon grid, including $\varepsilon = 2$ and $\varepsilon = 3$. The size of this gap varies with the elasticity, but the qualitative conclusion that coercion reduces long-run mechanization is unchanged. The lowest elasticity values generate a substantially poorer fit to the targeted moments. The sufficient monotonicity condition $a + 1 - \beta_c > 0$ holds at all reported values with $\varepsilon \geq 5$. When this condition fails, numerical checks still find one interior stationary root of the equilibrium condition, as the monotonicity condition is not necessary.

F Two-Household Extension

In the main model, I use a representative household to keep the household side simple and focus on the effects of coercion on aggregate growth. Here I separate the households. This allows me to show explicitly who receives labor income under coercion and how consumption differs across households. The static allocation and BGP conditions remain the same; only the transition changes, but quantitatively, the transition results also remain almost the same as in the baseline model.

Households and income. There are two households with fixed labor supplies H and L. One household supplies high-skill labor, owns firms, and chooses consumption and saving. The other supplies low-skill labor and consumes its current income. Let $\theta_t = 1$ under coercion and $\theta_t = 0$ under free labor. Low-skill productivity is then $A_L\gamma(i)c(i)^{\theta_t}$, as in the two regimes of the main model. Under coercion, low-skill workers receive a share $s \in (0,1)$ of the payment for their labor, which I interpret as the subsistence share. The rest goes to the owner household. Under free labor, low-skill workers receive their full labor income. Thus,

$$C_{L,t} = (1 - \theta_t + s\theta_t)w_{L,t}L, \tag{F-0-1}$$

$$\dot{\mathcal{A}}_t + C_{H,t} = r_t\mathcal{A}_t + w_{H,t}H + \theta_t(1-s)w_{L,t}L, \tag{F-0-2}$$

where $\mathcal{A}_t$ denotes asset holdings. Subsistence is paid from the rental income, so it does not require an additional resource cost. Using a share allows subsistence consumption to grow with the economy.

The high-skill household maximizes discounted log consumption with discount rate ρ, subject to its budget and the usual transversality conditions. Its Euler equation is $\dot{C}_{H,t}/C_{H,t} = r_t - \rho$. The low-skill household consumes the amount it receives and has no saving decision.

Static allocation and balanced growth. Having the two types of households described above changes who receives income and saves, but it does not change firms' static production decisions or equilibrium factor prices for given technologies and labor supplies. The task-allocation conditions therefore remain the same:

$$w_{L,t}L = (\bar{I}_t - I_t)Y_t, \qquad w_{H,t}H = (1 - \bar{I}_t + \phi I_t)Y_t.$$

The stationary mechanization conditions are also unchanged, including the lower frontier under coercion. The aggregate resource constraint becomes $Y_t = C_{H,t} + C_{L,t} + R_t$, where R_t is research expenditure. On the BGP, labor income and research are constant shares of output. Consumption in both households grows at the same rate as output. The high-skill household's Euler equation again gives $r = \rho + g$. Thus,

$$(\tau^*)^{\alpha-1}(\rho + \tau^*) = \frac{1-\phi}{\alpha\kappa}, \qquad g = I^{ss}\tau^* \ln(1+\lambda),$$

where I^{ss} is the stationary frontier in the relevant regime. At the same parameters, stationary mechanization, innovation intensity, and long-run growth are therefore the same as in the main model.

Transition dynamics. The difference during the transition comes from the fact that low-skill workers consume all the income they receive. As tasks are reallocated, their income share changes, which also changes the consumption available to the saving household. Combining the resource constraint with research expenditure gives

$$\frac{C_{H,t}}{Y_t} = 1 - \kappa I_t \tau_t^{\alpha} - \kappa^m \dot{I}_t^{\alpha} - (1 - \theta_t + s\theta_t)(\bar{I}_t - I_t). \tag{F-0-3}$$

Unlike on the BGP, consumption need not grow at the same rate for both households during the transition. Using the high-skill household's Euler equation gives the revised equation for vertical innovation within each institutional regime:

$$(\alpha - 1)\frac{\dot{\tau}_t}{\tau_t} = \rho + \tau_t - \frac{1 - \phi}{\alpha\kappa\tau_t^{\alpha-1}} + \frac{\mathrm{d}}{\mathrm{d}t}\ln\left(\frac{C_{H,t}}{Y_t}\right), \tag{F-0-4}$$

where the consumption share is given by (F-0-3). The equations for I_t and v_t keep the same form, but their paths can also change because they are solved jointly with τ_t.

Quantitative comparison. I set the subsistence share to $s = 0.15$ and the Northern labor ratio to $L^N/H^N = 1$.[29] I keep the Southern effective-labor ratios at 1.04 before emancipation and 0.7072 afterward. The initial relative GDP-per-worker normalization and CES elasticity $\varepsilon = 5$ also remain unchanged. I then re-estimate the same eight parameters using the same nine historical moments. Table 18 compares these estimates with the benchmark. All eight estimates change by less than 0.02%. Keeping the original parameters also gives a very similar 1910 ratio, 0.345717.

Thus, having two households has little effect on the fitted aggregate results compared to the baseline with the representative household. The Online Appendix, Section D gives the full derivation and the detailed consumption and welfare comparisons.

[29] I approximate $s = 0.15$ by dividing the maintenance costs of enslaved workers, including food, medical care, and other expenses, reported by Ransom and Sutch (2001) by the average slave-hiring rate reported by Evans (1962). The ratio $L^N/H^N = 1$ is needed to compute the transition in this extension, whereas in the baseline model, aggregate transition can be solved without specifying it. But at the BGP equilibrium, mechanization frontier, innovation intensity, and growth rates are all unaffected. Increasing the ratio from 1 to 1.5 and re-estimating the model leaves the aggregate paths and parameter estimates almost unchanged. Thus, this ratio mainly affects the distribution of income and consumption within the Northern households, which is not the main focus of this paper and extension.

Table 18: Benchmark and Re-estimated Two-Household Model

	Benchmark	Two households	Difference (%)
Panel A. South–North GDP-per-worker ratio			
1800	0.718619	0.718619	+0.000
1860	0.508200	0.508206	+0.001
1865 (before abolition)	0.491592	0.491595	+0.001
1865 (after abolition)	0.371759	0.371759	+0.000
1900	0.348908	0.348880	-0.008
1910	0.345714	0.345678	-0.010
Panel B. Estimated parameters			
Free mechanization frontier, I^{**}	0.357615	0.357636	+0.006
Coercive mechanization frontier, I^{*}	0.163238	0.163207	-0.019
Stationary innovation intensity, τ^{*}	0.327190	0.327171	-0.006
Innovation-cost ratio, $\xi = \kappa^{m}/\kappa$	97.399684	97.404872	+0.005
Initial Northern mechanization, $I_{N,1800}$	0.193717	0.193716	-0.001
Initial Southern mechanization, $I_{S,1800}$	0.127515	0.127514	-0.001
Coercive stationary labor cutoff, $\bar{I}$	0.537525	0.537488	-0.007
Coercion gradient, β_c	3.485821	3.486212	+0.011

Notes: Both models fit the same nine moments; the extension fixes $s = 0.15$ and $L^N/H^N = 1$, with other external inputs unchanged. Difference is $100 \times (\text{two households/benchmark} - 1)$. The normalized sums of squared moment residuals are 0.000539970 and 0.000540724, respectively.

This extension also allows me to study how the welfare effects of emancipation differ across households through consumption. Figure 32 shows that low-skill workers' consumption rises immediately as they retain their labor income, while owners' consumption falls after losing appropriated income.

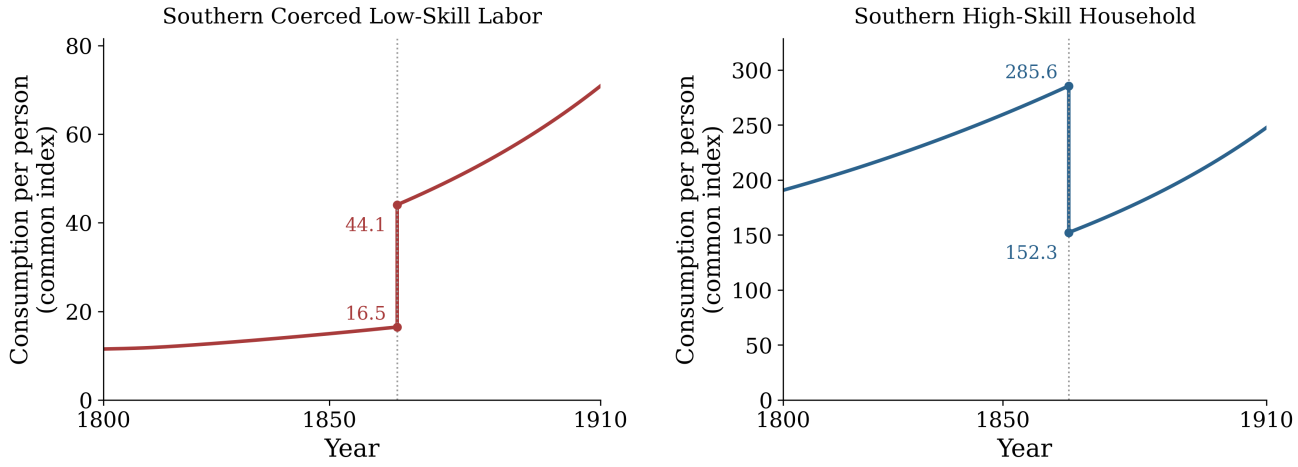


Figure 32: Consumption by Southern Household Type

Notes: Consumption per person, with population counts fixed at $N_H = 1$ and $N_L = 1.04$; Southern income per model worker in 1800 is normalized to 100. The vertical line marks emancipation in 1865. Low-skill workers are free thereafter. Direct coercion disutility is excluded.